Nonlocal-Unification-Informed Bayes Rule With Implications for Background Independence

A. Chawla

REAL Institute, Gurugram, India

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Schack, Brun, and Caves derived a quantum Bayes rule for exchangeable quantum states by showing that, under generalized measurements, the posterior state of unmeasured systems may be represented as a Bayesian update over density operators. The present article proposes a Nonlocal-Unification-informed extension of that rule. In Nonlocal Unification (NU), an observer is not only an abstract user of probabilities, but an embodied sampling function acting on a nonlocal informational substrate. The proposed update therefore acts on a joint model of quantum state and observer function,

$$p_A(\rho, \theta) \longmapsto p_A(\rho, \theta \mid k),$$

where ρ is the density-operator component of the model and θ parametrizes the observer's sampling architecture. The ordinary quantum Bayes rule is recovered when the sampling architecture is fixed. The article further examines NU information conservation, Frechet convergence of observer functions to a limiting observer, and a two-observer toy universe in which one observer reconstructs the internal spacetime of another. The resulting framework connects Bayesian inference, observer-relative spacetime reconstruction, and the informational field-equation side of NU.

Keywords: quantum Bayes rule; QBism; Nonlocal Unification; observer functions; exchangeability; information conservation; Frechet convergence; decoherence

1 Introduction

Bayesian methods enter quantum theory in a nontrivial way because the quantum state is not an ordinary classical random variable. Schack, Brun, and Caves showed that, when a prior state over several copies of a system is exchangeable, the posterior state assigned to unmeasured copies after a generalized measurement can be written as a Bayesian update over density operators [1]. This result gives a precise quantum analogue of Bayes' rule.

Quantum Bayesianism (QBism) develops the interpretive consequence of this viewpoint: the quantum state is an agent's probability assignment, not an observer-independent property of the system [2, 3]. Nonlocal Unification (NU), as developed in the author's related work, preserves this agent-centered orientation but supplements it with an informational ontology. In NU, an observer is an embodied sampling function acting on a nonlocal informational substrate; spacetime and objectivity arise from structured sampling and inter-observer consistency.

The purpose of this article is to formulate a first NU-informed quantum Bayes rule. The mathematical change is small but conceptually significant. Instead of assigning a prior only over candidate density operators ρ, an NU observer assigns a prior over pairs (ρ, θ), where θ describes the effective observer function or sampling architecture. The posterior is therefore

$$p_A(\rho, \theta \mid k) = \frac{p_A(k \mid \rho, \theta) p_A(\rho, \theta)}{p_A(k)}.$$

This rule reduces to the Schack-Brun-Caves update when θ is fixed. When θ is variable, the update concerns both the state-side model and the observer-side sampling structure.

The article is organized as follows. Section 2 reviews the Schack-Brun-Caves rule. Section 3 states the NU ingredients. Section 4 formulates the NU Bayes rule and its information-conserving form. Section 5 introduces NU exchangeability. Sections 6–8 present observer-function examples, including a limiting-observer construction and a two-observer decoherence scenario. Section 9 discusses the bridge to observer-relative field equations. Section 10 applies the framework to topological quantum computation.

2 Quantum Bayes Rule for Exchangeable States

Let $\mathcal{H}$ be the Hilbert space of a quantum system. Suppose an agent assigns an exchangeable prior state to $M + N$ copies:

$$\rho^{(M+N)} = \int p(\rho)\, \rho^{\otimes (M+N)} \, \mathrm{d}\rho.$$

Here $p(\rho)$ is a probability density over single-system density operators. A generalized measurement on M systems is represented by a POVM $\{E_k\}$, and the likelihood of outcome k given candidate state ρ is

$$p(k \mid \rho) = \mathrm{tr}\big(E_k \rho^{\otimes M}\big) .$$

The posterior density over ρ is then

$$p(\rho \mid k) = \frac{p(k \mid \rho) p(\rho)}{\int p(k \mid \rho) p(\rho) \, \mathrm{d}\rho}.$$

The posterior state of the remaining N systems is

$$\rho_k^{(N)} = \int p(\rho \mid k) \rho^{\otimes N} \, \mathrm{d}\rho.$$

This is the Schack-Brun-Caves quantum Bayes rule. Its key structural assumption is exchangeability: permutation invariance and extendibility of the prior over copies. This condition is what permits the posterior to be represented as a Bayesian update over density operators.

3 Observer Functions in Nonlocal Unification

NU modifies the ontology surrounding the inference problem. The quantum state remains agent-relative, but the agent is not treated as an abstract label. The agent is modeled as a sampling system.

Definition 1 (Nonlocal informational substrate)**.** *Let $\mathcal{I}$ denote a nonlocal informational substrate. It is not assumed to be embedded in spacetime. Observed spacetime structure is obtained through observer-dependent sampling of $\mathcal{I}$.*

Definition 2 (Observer function)**.** *An observer A is associated with a sampling function*

$$S_A = S(\theta_A) : \mathcal{I} \longrightarrow \mathcal{O}_A,$$

where θ_A denotes biological, neural, instrumental, and resolution parameters, and $\mathcal{O}_A$ is the observation space of A.

In this language, a measurement outcome is not simply the revelation of a pre-existing local property. It is an event in which an embodied observer samples the informational substrate under constraints that, at the operational quantum level, are represented by effective POVM elements.

4 The NU Bayes Rule

The ordinary quantum Bayes rule treats the candidate density operator as the variable of inference. NU allows the observer function itself to enter the inference problem. The prior is therefore a joint density

$$p_A(\rho, \theta),$$

where θ parametrizes the effective observer function. The likelihood is

$$p_A(k \mid \rho, \theta) = \operatorname{tr}\big(E_k(\theta)\rho^{\otimes M}\big)\,,$$

where $E_k(\theta)$ is the POVM element induced by the sampling architecture θ.

Proposition 1 (NU-informed quantum Bayes rule)**.** *Let A assign a joint prior $p_A(\rho, \theta)$ over density operators and observer-function parameters. If outcome k is represented by the effective POVM $\{E_k(\theta)\}$, then*

$$p_A(\rho, \theta \mid k) = \frac{\operatorname{tr}(E_k(\theta)\rho^{\otimes M})p_A(\rho, \theta)}{\iint \operatorname{tr}(E_k(\theta)\rho^{\otimes M})p_A(\rho, \theta)\ \mathrm{d}\rho\ \mathrm{d}\theta}.$$

The posterior state assigned to the remaining N systems is

$$\rho^{(N)}_{A,k} = \iint p_A(\rho, \theta \mid k)\rho^{\otimes N}\ \mathrm{d}\rho\ \mathrm{d}\theta.$$

If θ is fixed at θ_0 and $E_k(\theta_0) = E_k$, the rule reduces immediately to the Schack-Brun-Caves update. The new content is not a replacement of quantum Bayes, but an enlargement of the probability space.

5 Information Conservation

The NU framework treats information as fundamental and conserved across quantum, classical, and conscious processes [4]. Bayesian learning is therefore not interpreted as information appearing from nothing. The record, the observer's belief state, and the observer-accessible substrate description must be accounted for together.

Principle 1 (NU information conservation)**.** *For a measurement event experienced by observer A,*

$$\Delta I_A^{\text{belief}} + \Delta I_A^{\text{record}} + \Delta I_A^{\text{substrate}} = 0,$$

up to the chosen sign convention for informational flow.

One possible belief-side measure is

$$\Delta I_A^{\text{belief}} = D_{\text{KL}}(p_A(\rho, \theta \mid k) \,\|\, p_A(\rho, \theta)) \,.$$

The record contribution may be represented by the surprisal

$$I_A^{\text{record}}(k) = -\log p_A(k).$$

The substrate term is then the compensating term required by the chosen accounting. The central point is interpretive: NU-Bayes is not only an epistemic update, but an information-conserving sampling event.

6 NU Exchangeability

The Schack-Brun-Caves theorem requires exchangeability of the quantum state. NU requires the corresponding assumption relative to the observer function.

Definition 3 (NU exchangeability)**.** *A prior over $M + N$ systems and observer-function parameters is NU-exchangeable when it can be represented as*

$$p_A(\rho, \theta)\rho^{\otimes(M+N)}$$

and is permutation invariant over the sampled systems for fixed θ, while remaining extendible to larger numbers of systems under the same sampling architecture.

NU exchangeability preserves the operational role of exchangeability. It also admits observer-dependent likelihoods. Different observers, or the same observer under different sampling conditions, may assign different effective POVMs.

7 Example 1: A Single Observer with Uncertain Calibration

The following finite example isolates the difference between ordinary quantum Bayes and NU-Bayes.

Let the candidate qubit states be

$$\rho_0 = |0\rangle\langle 0|, \qquad \rho_1 = |1\rangle\langle 1|,$$

with prior probabilities $p(\rho_0) = p(\rho_1) = 1/2$. A computational basis measurement yields outcome $k = 0$.

Under ordinary quantum Bayes, the measurement is fixed:

$$p(k = 0 \mid \rho_0) = 1, \qquad p(k = 0 \mid \rho_1) = 0.$$

Hence

$$p(\rho_0 \mid k = 0) = 1, \qquad p(\rho_1 \mid k = 0) = 0.$$

Only the state assignment changes.

Now introduce two possible observer-function settings:

$$\theta_c = \text{correct calibration}, \qquad \theta_f = \text{flipped calibration}.$$

The likelihoods are

$$p(k = 0 \mid \rho_0, \theta_c) = 1, \quad p(k = 0 \mid \rho_1, \theta_c) = 0,$$

$$p(k = 0 \mid \rho_0, \theta_f) = 0, \quad p(k = 0 \mid \rho_1, \theta_f) = 1.$$

With uniform joint prior

$$p(\rho_i, \theta_j) = \frac{1}{4}, \qquad i \in \{0, 1\}, \quad j \in \{c, f\},$$

the normalizing probability is

$$p_A(k = 0) = \frac{1}{2}.$$

Thus

$$p_A(\rho_0, \theta_c \mid k = 0) = \frac{1}{2}, \qquad p_A(\rho_1, \theta_f \mid k = 0) = \frac{1}{2},$$

and the other two joint possibilities have posterior probability zero.

The ordinary rule asks: given this measurement, which state should be inferred? NU-Bayes asks: given this record, which state-and-observer pair should be inferred? If θ_c is later verified, NU-Bayes collapses to the ordinary answer. If θ_f is verified, the same record supports the opposite state.

8 Frechet-Convergent Observer Sequences

The second article in *Prajnanabha* and the later article on limit observers discuss observer functions as members of sequences converging to a limiting observer in a Frechet sense [4, 5]. In full form, this requires convergence of functions and their relevant derivatives on compact domains. The following toy version represents such convergence by decreasing sampling error.

Let

$$S_n = S(\theta_n), \qquad n = 1, 2, \ldots,$$

and suppose

$$S_n \longrightarrow S_\infty$$

in the observer-function topology. In the two-state model, write

$$p(k = 0 \mid \rho_0, \theta_n) = 1 - \epsilon_n,$$

$$p(k = 0 \mid \rho_1, \theta_n) = \epsilon_n,$$

where $\epsilon_n \to 0$. For a fixed θ_n,

$$p(\rho_0 \mid k = 0, \theta_n) = 1 - \epsilon_n, \qquad p(\rho_1 \mid k = 0, \theta_n) = \epsilon_n.$$

Therefore

$$\lim_{n\to\infty} p(\rho_0 \mid k = 0, \theta_n) = 1.$$

NU-Bayes can also update over the observer index:

$$p_A(\rho_i, \theta_n \mid k = 0) = \frac{p(k = 0 \mid \rho_i, \theta_n) p_A(\rho_i) p_A(\theta_n)}{\sum_m \sum_{j=0}^{1} p(k = 0 \mid \rho_j, \theta_m) p_A(\rho_j) p_A(\theta_m)}.$$

Repeated records consistent with the limiting observer concentrate posterior mass near observer functions with small ϵ_n, and in the ideal limit near (ρ_0, θ_∞).

Ordinary quantum Bayes can produce a convergent sequence of posteriors if a convergent sequence of POVMs $E_k^{(n)} \to E_k^{(\infty)}$ is supplied externally. It does not, however, infer the limiting observer as a variable of the model. NU-Bayes internalizes the observer sequence by assigning probability to θ_n itself.

9 Example 2: Two Observer Functions and Decoherence

Consider a universe containing only two observer functions:

$$\{\gamma_1, \gamma_2\}.$$

Each may be embedded in a Frechet-convergent observer sequence and is therefore affiliated with the NU-limit. In this toy universe, however, the relation between γ_1 and γ_2 exhausts the relational structure under consideration.

Suppose γ_2 treats γ_1 as an object of inference. It receives a record R_{21} generated through interaction with γ_1 and applies NU-Bayes:

$$p_{\gamma_2}(\alpha_1, \gamma_1 \mid R_{21}).$$

Here α_1 is the informational state sampled by γ_1. If the posterior concentrates, γ_2 has inferred that

$$\gamma_1 \text{ observing } \alpha_1$$

is the generative explanation of the record.

In NU this is a spacetime reconstruction statement. Since an observer function maps informational distinctions into an internal geometry, γ_2 can reconstruct

$$\Phi_{\mu\nu}^{(\gamma_1)} = \gamma_1[\Delta I_{\alpha_1}],$$

the spacetime or informational geometry generated by γ_1.

This reconstruction is an information gain for γ_2:

$$\Delta I_{\gamma_2}^{\text{gain about } \gamma_1} > 0.$$

NU conservation requires a compensating accounting. In a two-observer universe there is no third reservoir, so the balance is relational:

$$\Delta I_{\gamma_2}^{\text{gain about } \gamma_1} + \Delta I_{\gamma_2}^{\text{loss}} + \Delta I_{\gamma_1}^{\text{return}} = 0.$$

The loss term should not be read as ordinary forgetting. It denotes a loss of available informational freedom or coherence. Before the reconstruction, γ_2 may stand in a less determinate relation to γ_1. After reconstruction, a definite relational record has been selected. The space of coherent possibilities available to γ_2 has narrowed.

This gives an NU interpretation of decoherence:

$$\begin{gathered}
\gamma_2 \text{ samples } \gamma_1, \\
\Downarrow \\
\gamma_2 \text{ gains information about } \gamma_1, \\
\Downarrow \\
\gamma_2 \text{ loses coherent informational freedom}, \\
\Downarrow \\
\text{the compensating information returns to the } \gamma_1 \text{ side of the relation.}
\end{gathered}$$

Thus decoherence is the loss of observer-side informational freedom caused by turning another observer function into an object of determinate reconstruction.

10 Bridge to Informational Field Equations

The preceding example shows why NU-Bayes is not merely a rule for belief revision. A posterior pair such as

$$(\alpha, \gamma)$$

is a reconstruction key: it states that observer function γ, sampling informational state α, is the generative explanation of the record. From this pair one reconstructs the internal milieu of

γ:

$$\Phi^{(\gamma)}_{\mu\nu} = \gamma[\Delta I_\alpha].$$

This object is the entry point to the informational field-equation side of NU. Schematically,

$$\Phi^{(\gamma)}_{\mu\nu} = \gamma[\Delta I_\alpha] \quad \Longrightarrow \quad G^{(\gamma)}_{\mu\nu} = \mathcal{F}_{\mu\nu}\left(\Delta I_\alpha, \gamma, \partial\gamma, \partial\Delta I_\alpha, \ldots\right).$$

The detailed form of $\mathcal{F}_{\mu\nu}$ belongs to the field theory of NU. The role of NU-Bayes is to supply the inferred ingredients: informational content and observer function.

Ordinary quantum Bayes stops at a state assignment relative to a fixed measurement structure. NU-Bayes goes further by inferring the observer-function component. This is what makes the update generative: it can be used to reconstruct an observer-relative spacetime and then connect to informational field equations.

11 Discussion

The proposed rule is conservative at the operational quantum level. It does not alter the Born rule. Rather, it expands the probabilistic model to include the sampling architecture through which the Born-rule likelihood is expressed. The standard quantum Bayes rule is obtained when this architecture is fixed.

The additional NU claims are interpretive and structural. First, the observer is treated as a physically instantiated sampling function. Second, Bayesian learning is embedded in an information-conservation principle. Third, observer functions may be organized into Frechet-convergent sequences affiliated with a limiting observer. Fourth, inference of an observer function can serve as a bridge to reconstruction of observer-relative spacetime.

The two-observer toy model illustrates a relational account of decoherence. If γ_2 reconstructs γ_1, then γ_2 gains information about γ_1's generated spacetime. Conservation requires a compensating loss of γ_2's own coherent informational freedom, with the balance returned into the γ_1–γ_2 relation.

12 Example 3: Topological Quantum Computation

We now give a non-toy example using topological quantum computation (TQC). The purpose is to show that the NU-Bayes formalism can be applied to a standard quantum-information pipeline: state preparation, topological encoding, braid evolution, noisy quantum channel, readout, and Bayesian reconstruction. This follows the standard TQC picture in which non-Abelian anyons store information nonlocally, braid operations implement gates, and fusion measurements read out the encoded information [7].

12.1 Computational Hilbert space

Use Fibonacci anyons, whose topological charge set is

$$\{1, \tau\}, \qquad \tau \times \tau = 1 + \tau.$$

The quantum dimension of τ is the golden ratio

$$\varphi = \frac{1+\sqrt{5}}{2}.$$

Consider four τ anyons with total charge 1. The associated fusion space is two-dimensional. We identify it with a logical qubit:

$$\mathcal{H}_{\text{top}} = \text{span}\{|0_L\rangle, |1_L\rangle\} \cong \mathbb{C}^2.$$

The computational basis is defined by the fusion outcome of the first two anyons:

$$|0_L\rangle : \quad (\tau\tau) \to 1, \qquad |1_L\rangle : \quad (\tau\tau) \to \tau.$$

Thus a conventional quantum-information state

$$\alpha = |\psi\rangle\langle\psi|, \qquad |\psi\rangle = a|0\rangle + b|1\rangle, \quad |a|^2 + |b|^2 = 1,$$

is encoded by the isometry

$$V : \mathbb{C}^2 \to \mathcal{H}_{\text{top}}, \qquad V|0\rangle = |0_L\rangle, \quad V|1\rangle = |1_L\rangle.$$

The encoded state is

$$\alpha_{\text{top}} = V\alpha V^\dagger.$$

12.2 Topological encoding and braid representation

Let B_4 be the braid group on four strands with generators $\sigma_1, \sigma_2, \sigma_3$. In the Fibonacci theory, these generators act on $\mathcal{H}_{\text{top}}$ by a unitary representation

$$\rho_{\text{Fib}} : B_4 \to U(\mathcal{H}_{\text{top}}).$$

In a standard fusion basis, the elementary data are the R and F matrices

$$R^1_{\tau\tau} = e^{-4\pi i/5}, \qquad R^\tau_{\tau\tau} = e^{3\pi i/5},$$

and

$$F = \begin{pmatrix} \varphi^{-1} & \varphi^{-1/2} \\ \varphi^{-1/2} & -\varphi^{-1} \end{pmatrix}.$$

For example, braiding the first two anyons gives

$$\rho_{\text{Fib}}(\sigma_1) = \begin{pmatrix} R^1_{\tau\tau} & 0 \\ 0 & R^\tau_{\tau\tau} \end{pmatrix},$$

while braiding adjacent anyons in a different fusion channel is obtained by changing basis with F, applying R, and changing back. For instance,

$$\rho_{\mathrm{Fib}}(\sigma_2) = F \begin{pmatrix} R^1_{\tau\tau} & 0 \\ 0 & R^\tau_{\tau\tau} \end{pmatrix} F^{-1}.$$

For a braid word

$$b = \sigma_{i_m}^{s_m} \cdots \sigma_{i_1}^{s_1}, \qquad s_j \in \{\pm 1\},$$

the implemented logical unitary is

$$U_b = \rho_{\mathrm{Fib}}(b).$$

This formulation is topological because U_b depends on the braid class, not on the metric details of the anyon trajectories. The same representation is connected to knot and link invariants: the Markov trace of the braid representation, after closure of the braid, gives the Jones polynomial at a root of unity associated with the Fibonacci theory. Equivalently, in the bracket formulation, the Kauffman bracket of the braid closure evaluates the same topological data after normalization. Thus the logical gate is encoded in the topology of a braid, while the associated link invariant records the same topological equivalence class from the knot-theoretic side.

12.3 A CPTP map for topological noise

Let the intended braid be b. The ideal encoded output is

$$\beta_b = U_b \alpha_{\mathrm{top}} U_b^\dagger.$$

A physically realistic TQC device is not perfectly isolated. A common topological error mechanism is the creation of an anyon–anti-anyon pair from the vacuum, accidental braiding around part of the encoded system, and subsequent annihilation. Such an event is topological because its effect depends on the braid class of the error loop.

Model the implemented operation by the CPTP map

$$\mathcal{E}_{b,\eta}(\rho) = (1-\eta) U_b \rho U_b^\dagger + \eta \sum_\ell q_\ell W_\ell U_b \rho U_b^\dagger W_\ell^\dagger,$$

where

$$0 \le \eta \le 1, \qquad q_\ell \ge 0, \qquad \sum_\ell q_\ell = 1,$$

and each $W_\ell = \rho_{\mathrm{Fib}}(e_\ell)$ is the unitary associated with an error braid e_ℓ. This is completely positive and trace preserving because it is a convex mixture of unitary conjugations.

The output state before readout is

$$\beta = \mathcal{E}_{b,\eta}(\alpha_{\mathrm{top}}).$$

12.4 Fusion readout and quantum Bayes reconstruction

Readout is performed by measuring a fusion outcome. More generally, let

$$\{M_y\}_{y\in Y}$$

be an informationally complete POVM on $\mathcal{H}_{\text{top}}$. For a candidate initial state α and known channel $\mathcal{E}_{b,\eta}$, the likelihood of a readout outcome y is

$$p(y \mid \alpha, b, \eta) = \operatorname{tr}\Big[M_y\, \mathcal{E}_{b,\eta}(V\alpha V^\dagger)\Big] \,.$$

Given a prior density $p(\alpha)$ over input qubit states, quantum Bayes gives

$$p(\alpha \mid y, b, \eta) = \frac{\operatorname{tr}[M_y\, \mathcal{E}_{b,\eta}(V\alpha V^\dagger)]p(\alpha)}{\int \operatorname{tr}[M_y\, \mathcal{E}_{b,\eta}(V\alpha V^\dagger)]p(\alpha)\, \mathrm{d}\alpha}.$$

For a dataset

$$D = \{y_1, \dots, y_L\}$$

of independent repetitions using the same braid and channel, the likelihood is

$$p(D \mid \alpha, b, \eta) = \prod_{r=1}^{L} \operatorname{tr}\Big[M_{y_r}\, \mathcal{E}_{b,\eta}(V\alpha V^\dagger)\Big] \,.$$

The posterior is

$$p(\alpha \mid D, b, \eta) = \frac{p(D \mid \alpha, b, \eta)p(\alpha)}{\int p(D \mid \alpha, b, \eta)p(\alpha)\, \mathrm{d}\alpha}.$$

The reconstructed initial quantum information may be represented by the posterior mean

$$\widehat{\alpha} = \int \alpha\, p(\alpha \mid D, b, \eta)\, \mathrm{d}\alpha.$$

If the POVM is informationally complete and the channel is known and invertible on the code subspace in the statistical sense, the posterior concentrates around the true input state in the large-data limit.

12.5 NU-Bayes version of the TQC chain

NU-Bayes extends the preceding reconstruction by including the observer function. In a TQC experiment, the observer function is not merely a human observer. It includes the entire sampling architecture:

$$\gamma_{\text{TQC}} = \{\text{fusion basis, braid control, interferometer, readout, calibration, decoding map}\}.$$

The NU prior is therefore a joint prior

$$p_A(\alpha, \gamma, b, \eta),$$

where γ parametrizes the observer-function structure by which topological records are sampled and decoded. The NU likelihood is

$$p_A(D \mid \alpha, \gamma, b, \eta) = \prod_{r=1}^{L} \mathrm{tr}\Big[M_{y_r}(\gamma)\, \mathcal{E}_{b,\eta}^{(\gamma)}\big(V_\gamma \alpha V_\gamma^\dagger\big)\Big].$$

The NU posterior is

$$p_A(\alpha, \gamma, b, \eta \mid D) = \frac{p_A(D \mid \alpha, \gamma, b, \eta) p_A(\alpha, \gamma, b, \eta)}{\int p_A(D \mid \alpha, \gamma, b, \eta) p_A(\alpha, \gamma, b, \eta)\, \mathrm{d}\alpha\, \mathrm{d}\gamma\, \mathrm{d}b\, \mathrm{d}\eta}.$$

This posterior does more than reconstruct the initial qubit state. It also reconstructs which observer-function architecture made the topological record intelligible. A high-posterior tuple

$$(\alpha, \gamma, b, \eta)$$

states that the initial quantum information α, encoded through the observer-function γ, braided by b, and passed through the topological channel with strength η, is the generative explanation of the observed fusion data D.

12.6 Topological invariants as observer records

The knot-theoretic side can be included explicitly. Let $\widehat{b}$ be the closure of the braid b, and let

$$J_{\widehat{b}}(q)$$

denote the Jones polynomial evaluated at the Fibonacci root of unity associated with $q = e^{2\pi i/5}$, or equivalently let

$$\langle \widehat{b} \rangle_A$$

denote the corresponding Kauffman bracket evaluation before normalization. A topological observer may include such an invariant in its record:

$$R = (D, J_{\widehat{b}}(q)).$$

Then NU-Bayes may use

$$p_A(\alpha, \gamma, b, \eta \mid R)$$

rather than only $p_A(\alpha, \gamma, b, \eta \mid D)$. The invariant does not replace the Hilbert-space quantum data; it constrains the topological equivalence class of the braid used to generate the quantum channel. In this way, topological data and quantum readout data enter the same Bayesian reconstruction.

12.7 Information conservation and generated spacetime

In ordinary quantum Bayes, the end product is the reconstructed initial state $\widehat{\alpha}$. In NU-Bayes, the end product is a posterior over generative tuples. If the posterior concentrates near

$$(\alpha_*, \gamma_*, b_*, \eta_*),$$

then the statement is:

$$\gamma_* \text{ sampling } \alpha_* \text{ through braid } b_* \text{ and channel } \eta_* \text{ generated } R.$$

From the NU viewpoint, γ_* also generates an internal spacetime or informational geometry from the sampled informational content:

$$\Phi^{(\gamma_*)}_{\mu\nu} = \gamma_*[\Delta I_{\alpha_*, b_*, \eta_*}].$$

The information gained by reconstructing the initial quantum state, the braid class, and the observer-function architecture must be balanced by the NU conservation law:

$$\Delta I_A^{\text{state}} + \Delta I_A^{\text{topology}} + \Delta I_A^{\text{observer}} + \Delta I_A^{\text{record}} + \Delta I_A^{\text{substrate}} = 0.$$

Thus, in a topological quantum computer, NU-Bayes treats the observed fusion outcomes and knot/link invariants as records through which an observer reconstructs not only the encoded quantum information, but also the topological channel and the observer-relative sampling geometry that made the information available.

12.8 Numerical simulation and NU spacetime visualization

To make the preceding construction empirical in the minimal numerical sense, we simulated the full chain in Python. The script

```
scripts/simulate_tqc_nubayes.py
```

implements the following pipeline:

$$\begin{gathered}\text{initial qubit state } \alpha \\ \Downarrow \\ \text{Fibonacci-anyon encoding } V\alpha V^\dagger \\ \Downarrow \\ \text{braid unitary } U_b \\ \Downarrow \\ \text{CPTP topological braid-error channel } \mathcal{E}_{b,\eta} \\ \Downarrow \\ \text{tetrahedral POVM readout} \\ \Downarrow \\ \text{Q-Bayes and NU-Bayes reconstruction} \\ \Downarrow \\ \text{NU informational-spacetime reconstruction.}\end{gathered}$$

The simulated braid word was

$$b = \sigma_1\sigma_2\sigma_1^{-1}\sigma_2\sigma_1,$$

with topological error strength

$$\eta = 0.08.$$

The true NU observer-function visibility parameter was set to

$$\gamma_{\text{true}} = 0.86.$$

The true input state had Bloch vector

$$r_{\text{true}} = (0.3807,\ 0.6924,\ -0.6129).$$

After 360 synthetic readout shots, the empirical tetrahedral-POVM counts were

$$(74,\ 110,\ 148,\ 28).$$

Quantum Bayes reconstruction, with the braid channel and observer visibility treated as known, returned

$$\widehat{r}_{\text{Q}} = (0.1508,\ 0.8639,\ -0.4558),$$

with fidelity

$$F_{\text{Q}} = 0.9675.$$

NU-Bayes reconstruction, with observer visibility inferred jointly with the initial state, returned

$$\widehat{r}_{\text{NU}} = (0.1351,\ 0.8665,\ -0.4547),$$

with fidelity

$$F_{\text{NU}} = 0.9650,$$

and posterior mean

$$\hat{\gamma} = 0.8782.$$

The final information gain, measured as the KL divergence of the posterior from the prior over the initial-state grid, was

$$4.1808 \text{ nats}.$$

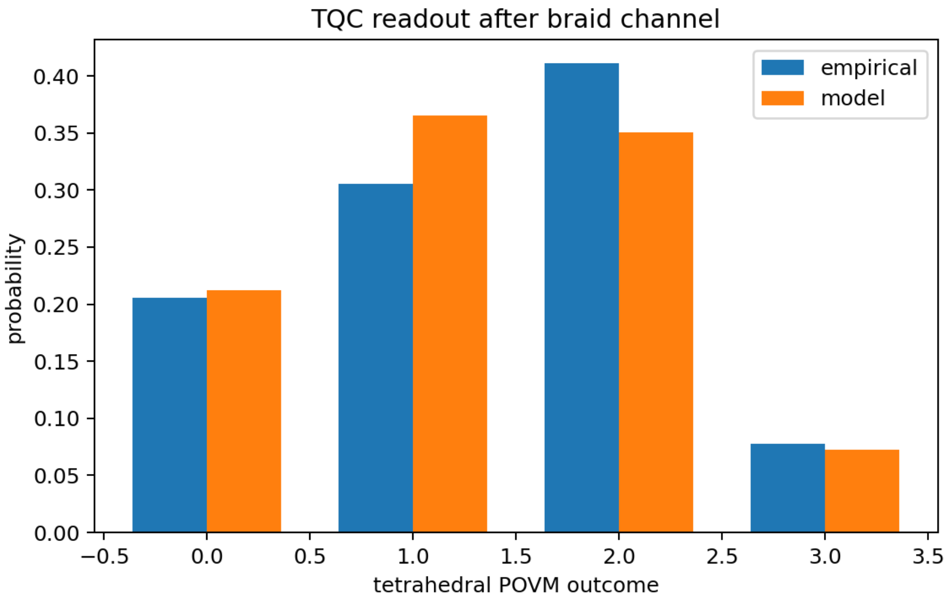

Figure 1: Synthetic empirical readout frequencies from the topological braid channel, compared with the model probabilities.

The NU-specific step is the reconstruction of an informational spacetime from the posterior information curve. Let

$$I_t = D_{\mathrm{KL}}(p_t(\alpha) \,\|\, p_0(\alpha))$$

be the information gained after the first t readout shots. In the simulation, the inferred observer parameter $\hat{\gamma}$ is used to define a simple observer-generated clock field

$$\tau(t, x) = t + 0.35\,\hat{\gamma}\,\frac{I_t}{\max_s I_s}\exp\left[-\left(\frac{x}{0.45}\right)^2\right],$$

and a conformal informational metric factor

$$\Omega(t, x) = 1 + 0.70\,\hat{\gamma}\,\frac{I_t}{\max_s I_s}\exp\left[-\left(\frac{x}{0.45}\right)^2\right].$$

These are not proposed as universal NU field equations. They are a minimal visualization of the reconstruction principle: as the observer gains information about the topologically encoded input state and the sampling architecture, the observer-relative internal spacetime is deformed by the accumulated informational content.

The empirical lesson of the simulation is modest but concrete. Q-Bayes reconstructs the initial quantum information from the observed output of a known topological channel. NU-Bayes

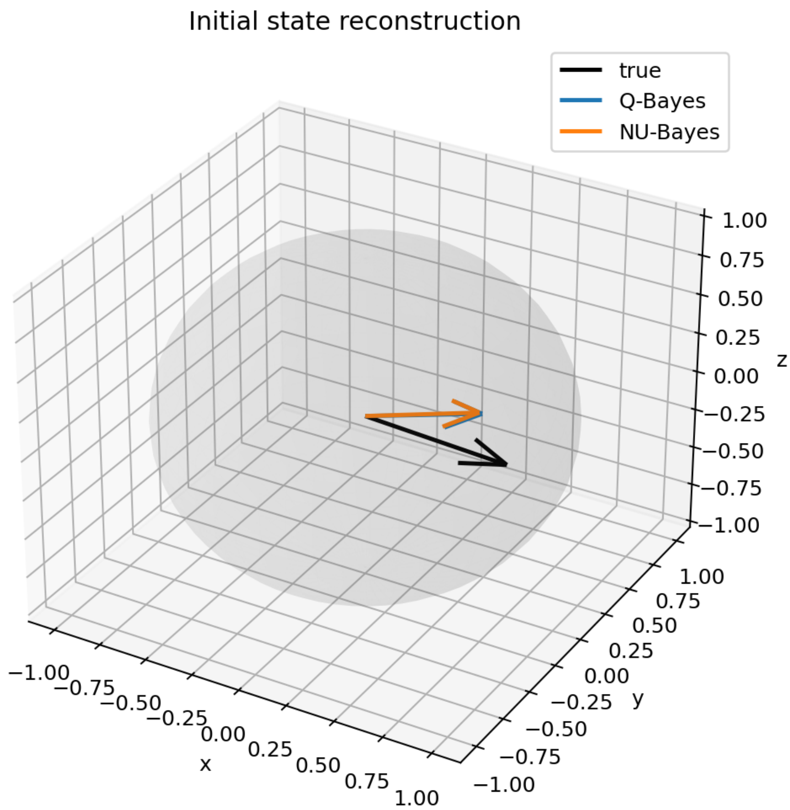

Figure 2: Initial-state reconstruction on the Bloch sphere. The Q-Bayes and NU-Bayes estimates are close to the true input state after the simulated topological channel and readout.

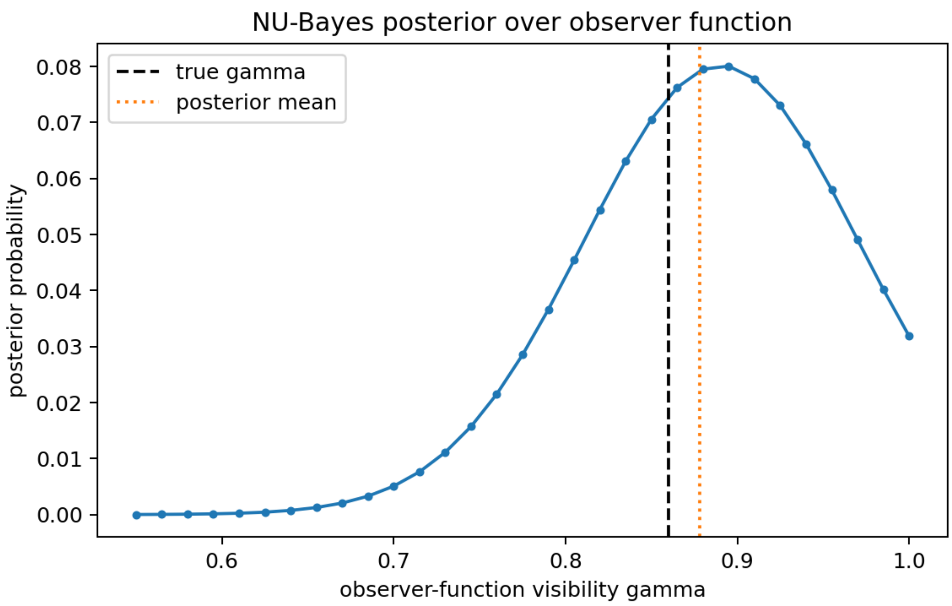

Figure 3: NU-Bayes posterior over the observer-function visibility parameter γ. The posterior mean is close to the true simulated value.

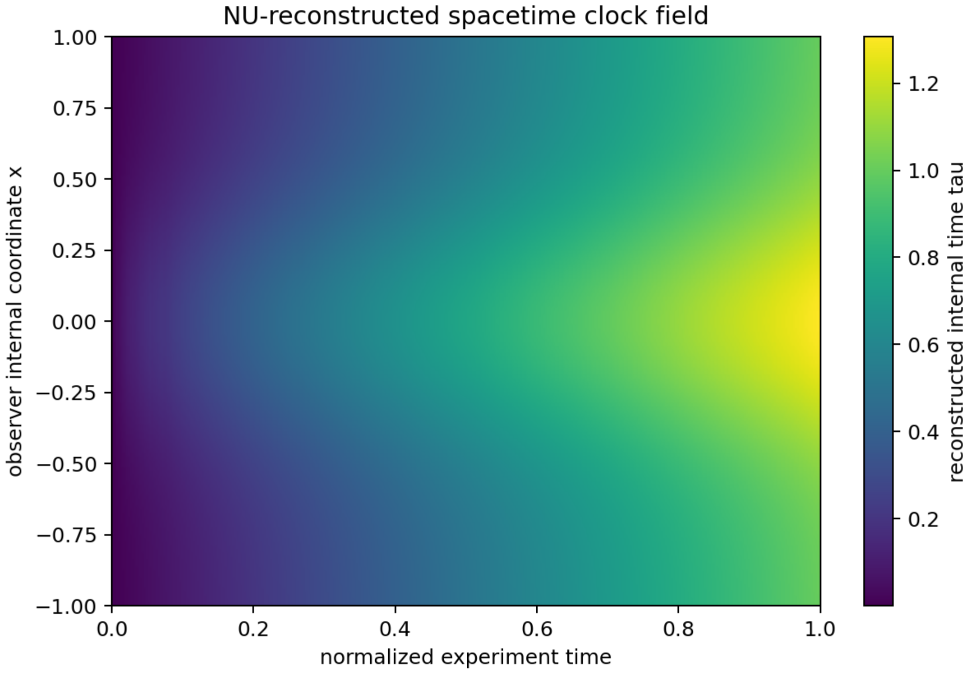

Figure 4: NU-reconstructed internal clock field $\tau(t, x)$ from the posterior information curve of the simulated topological quantum computation.

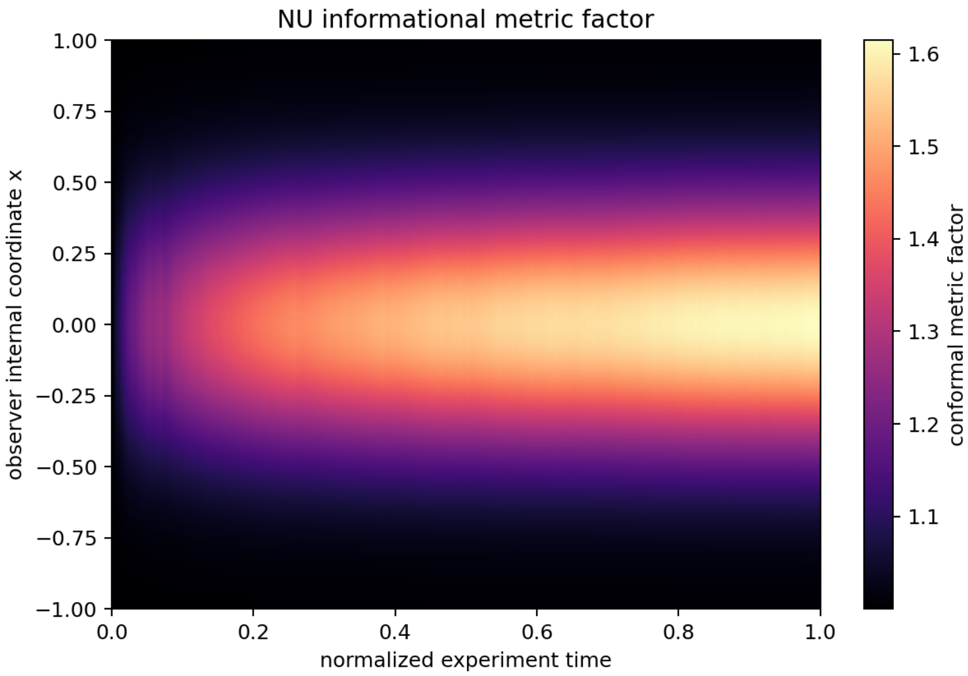

Figure 5: NU informational metric factor $\Omega(t, x)$ generated from the same reconstruction. The deformation tracks accumulated Bayesian information gain.

performs the same state-reconstruction task while also inferring part of the observer-function structure. The inferred observer function then supplies the parameter needed to visualize an observer-relative informational spacetime generated by the reconstruction process.

13 Jones-Polynomial Observables, QFT, and Generated Curved Spacetime

The preceding example used braid words, braid closures, and Jones-polynomial data as topological records supplementing the quantum readout data. This section makes that bridge explicit and connects it to the quantum-field-theoretic side of topological quantum computation.

Witten's construction of the Jones polynomial from quantum field theory uses Chern-Simons theory. For a compact gauge group G and connection A on a three-manifold M, the Chern-Simons action is

$$S_{\mathrm{CS}}[A] = \frac{k}{4\pi} \int_M \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right).$$

Given a link $L \subset M$, the corresponding Wilson-loop observable is

$$W_R(L) = \mathrm{Tr}_R\, \mathcal{P} \exp\left(\oint_L A \right),$$

where R is a representation and $\mathcal{P}$ denotes path ordering. The Chern-Simons path integral assigns expectation values

$$\langle W_R(L) \rangle_{\mathrm{CS}} = \frac{\int \mathcal{D}A\, W_R(L) \exp(iS_{\mathrm{CS}}[A])}{\int \mathcal{D}A\, \exp(iS_{\mathrm{CS}}[A])}.$$

For suitable choices of G, R, and the level k, these expectation values reproduce the Jones polynomial, or closely related colored Jones invariants, evaluated at roots of unity. Thus the observable of the quantum field theory is simultaneously a topological invariant of the link.

For the present purpose, we use the following simplifying assumption. Let $\mathcal{O}_{\mathrm{top}}$ denote a class of observed topological objects, such as braid closures or links. Assume a homeomorphism, at the level of observed objects rather than observer functions,

$$\chi : \mathcal{O}_{\mathrm{top}} \longrightarrow \mathcal{J},$$

where $\mathcal{J}$ is the corresponding space of Jones-polynomial observables. Thus an observed link L is represented by

$$\chi(L) = J_L(q),$$

with q determined by the Chern-Simons level. This assumption is not meant to assert that the Jones polynomial distinguishes all links in complete mathematical generality. It is a controlled modeling assumption for the present NU reconstruction: the observed object is identified with its Jones-polynomial observable within the restricted class under consideration.

13.1 NU sampling of Jones-polynomial observables

Let γ be an observer function whose input is a Jones-polynomial observable. Then

$$\gamma : \mathcal{J} \longrightarrow \mathcal{O}_\gamma.$$

If the observed topological object is L, then the sampled informational content is

$$\Delta I_L = \Delta I[J_L(q)].$$

The NU-generated spacetime associated with this observation is

$$\Phi_{\mu\nu}^{(\gamma,L)} = \gamma[\Delta I[J_L(q)]].$$

This is the same reconstruction principle used in the Bayesian topological-computation example, now expressed directly at the level of field-theoretic observables. The Jones polynomial is not merely a classical label attached after the quantum computation. It is an observable that can be sampled by γ, and that sampling generates an observer-relative spacetime.

The resulting chain is

$$L \xrightarrow{\chi} J_L(q) \overset{\text{Witten}}{\longleftrightarrow} \langle W_R(L)\rangle_{\text{CS}} \xrightarrow{\gamma} \Phi_{\mu\nu}^{(\gamma,L)}.$$

Thus a topological quantum-field observable is embedded into a generated spacetime by the observer function. This gives a novel link: quantum fields and spacetime are related not by assuming a background geometry first, but by having an observer function sample a field-theoretic invariant and generate the geometry in which that sampled content is experienced.

13.2 Comparison with quantum field theory in curved spacetime

Traditional quantum field theory in curved spacetime begins with a pseudo-Riemannian manifold $(M, g_{\mu\nu})$ and then quantizes a field on that fixed classical background. For example, a real scalar field ϕ on a curved spacetime has action

$$S[\phi; g] = -\frac{1}{2}\int_M \left(g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + m^2\phi^2 + \xi R\phi^2\right)\sqrt{-g}\, d^4x.$$

The corresponding field equation is

$$(\Box_g + m^2 + \xi R)\phi = 0.$$

The background metric $g_{\mu\nu}$ is supplied first. The quantum field is then studied relative to that supplied geometry. Particle creation, Hawking radiation, and the Unruh effect all arise from the interplay between quantum fields and nontrivial spacetime structure.

The NU link reverses the explanatory order. The observed quantum-field object is first represented by an invariant,

$$J_L(q) \sim \langle W_R(L)\rangle_{\text{CS}},$$

and the observer function then generates a metric-like structure:

$$g_{\mu\nu}^{(\gamma,L)} := \Phi_{\mu\nu}^{(\gamma,L)} = \gamma[\Delta I[J_L(q)]].$$

If this generated metric is pseudo-Riemannian, then one may formulate a field equation on the generated spacetime:

$$(\Box_{g^{(\gamma,L)}} + m^2 + \xi R[g^{(\gamma,L)}])\phi = 0.$$

The difference is conceptual. In ordinary QFT in curved spacetime, $g_{\mu\nu}$ is an external classical input. In the NU reformulation, $g_{\mu\nu}^{(\gamma,L)}$ is reconstructed from the observer's sampling of a quantum-field/topological observable.

13.3 Example: scalar field on a generated Robertson-Walker geometry

As a concrete comparison, consider the standard example of a scalar field on a spatially flat Robertson-Walker spacetime:

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2.$$

In conformal time η, this becomes

$$ds^2 = a(\eta)^2(-d\eta^2 + d\mathbf{x}^2).$$

For a scalar field mode $\phi_{\mathbf{k}}(\eta, \mathbf{x}) = a(\eta)^{-1}\chi_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}$, the mode equation is

$$\chi_k'' + \left[k^2 + m^2 a(\eta)^2 + (6\xi - 1)\,\frac{a''(\eta)}{a(\eta)}\right]\chi_k = 0.$$

In the traditional treatment, the scale factor $a(\eta)$ is supplied as part of the background spacetime. In the NU version, take the scale factor to be generated from the sampled Jones-polynomial information:

$$a_{\gamma,L}(\eta) = 1 + \lambda\,\Gamma_\gamma(\eta)\,\mathcal{I}[J_L(q)],$$

where λ is a coupling scale, $\Gamma_\gamma(\eta)$ is the observer-function response profile, and $\mathcal{I}[J_L(q)]$ is an information measure assigned to the Jones-polynomial observable. For example, one may use a normalized complexity or surprisal functional of the coefficient vector of $J_L(q)$.

The NU-reformulated scalar mode equation is then

$$\chi_k'' + \left[k^2 + m^2 a_{\gamma,L}(\eta)^2 + (6\xi - 1)\,\frac{a_{\gamma,L}''(\eta)}{a_{\gamma,L}(\eta)}\right]\chi_k = 0.$$

This is formally the same type of equation as in QFT in curved spacetime, but the curvature term is now traced to observer sampling of a topological quantum-field observable. Particle creation, when it occurs, is then interpreted as a consequence of the time-dependence of the observer-generated spacetime rather than of a background supplied independently of observation.

13.4 Bayesian reconstruction of the field-spacetime link

The NU-Bayes posterior for this setting is

$$p_A(\alpha, \gamma, L, k_{\text{CS}} \mid R),$$

where R contains quantum readout data and the topological invariant record,

$$R = (D, J_L(q)).$$

A concentrated posterior near

$$(\alpha_*, \gamma_*, L_*, k_*)$$

states that the initial quantum information α_*, sampled by observer function γ_* through the Chern-Simons/Jones observable associated with L_*, generated the record R. From this posterior one reconstructs

$$g_{\mu\nu}^{(\gamma_*, L_*)} = \gamma_*[\Delta I[J_{L_*}(q_*)]],$$

and then studies the corresponding quantum field equation on that generated curved manifold.

This gives the desired bridge:

$$\text{topological observable} \longrightarrow \text{NU-generated spacetime} \longrightarrow \text{QFT on generated curved spacetime.}$$

The bridge differs from traditional QFT in curved spacetime because the metric is no longer primitive. It is reconstructed from the observer-function sampling of a quantum-field invariant.

14 Conclusion

A NU-informed quantum Bayes rule has been proposed:

$$p_A(\rho, \theta \mid k) = \frac{p_A(k \mid \rho, \theta) p_A(\rho, \theta)}{p_A(k)}.$$

It reduces to the Schack-Brun-Caves rule when the observer function is fixed. When the observer function is variable, the posterior identifies not only the state component of the model but also the sampling architecture that explains the record. This allows NU-Bayes to address limiting observer sequences, observer-relative spacetime reconstruction, and relational decoherence. In this sense, NU-Bayes connects the Bayesian side of quantum inference with the informational field-equation side of Nonlocal Unification.

Declarations

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Data availability. No datasets were generated or analyzed. LLMs were used to produce the work.

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