Wigner Function Approach to Brain Decoherence Processes: a Proposal

Aman Chawla

January 18, 2024

Abstract

Wigner functions are quasi-probability distributions which can provide insight into the nature of quantum states in phase space. In this note, we bring attention to the open problem of redoing the derivations of [1] in terms of Wigner functions.

Quantum state space or Hilbert space is notoriously difficult to visualize completely. As the saying goes, "Hilbert space is a large space." This difficulty, one could think, might be transcended by going into a dual space. For as we know, dual spaces often have compact representations of signals as compared to the corresponding space. Take, for example, frequency and time. A signal spread widely in time has a compact support in the frequency domain and vice versa.

In [1], the author develops the equations for the density matrix of two-particle states that evolve through an interaction governed by a potential function. The evolution of the density matrix is difficult to visualize explicitly. One way to visualize it of course is to go to a finite dimensional Hilbert space and start with vectors for states and matrices for operations and work out an example. Going to high dimensions in this way could pose a problem. However, this is a sound method.

The other approach is to transform states to phase space. In such a setting, the density matrix is transformed via a Wigner integral. A Wigner distribution is not a probability distribution since it can go negative whereas a true p.d.f. is always greater than or equal to zero. The integral introduces a dummy variable of integration which is integrated out. As the variable ranges over its values, the bra sweeps out positional states monotonically as does the ket, though in the opposite sense. Thus the process is akin to ranging the density matrix over all possible positional combinations. In this sense, the entire density matrix is 'characterized.' Thus, everything encoded in a density matrix is available in the Wigner function. To go to higher dimensions extends the problem to one of multiple integrals instead of a single integral. A multiple integral is a sequence of single integrals performed, with the first one done in the setting where the rest of the variables are considered as constants and so on sequentially.

In the paper by Tegmark, the author sets up the ion-ion collision problem in terms of the density matrix. Tegmark himself suggests that a Wigner function

approach could be useful. One way it can help is that visualizing the position variables and their evolution becomes easier. Therefore one can decide on the best way to optimize collision parameters so as to obtain the best evolution path. The fine combed picture would also shed more light on the pertinent decoherence time.

References

[1] Max Tegmark. Importance of quantum decoherence in brain processes. *Physical review E*, 61(4):4194, 2000.