Vedic Approach to a Short Internode

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Abstract

In this brief paper, the authors present the Vedic solution of the discretized equation representing the internode of a myelinated axon.

The Vedic method for solving inhomogeneous simultaneous equations is adopted in this paper. The method makes use of the formula "samkalana-vyavakalanabhyam" presented in [1]. The equation to be solved is,

$$C\frac{\partial V(x,t)}{\partial t} = \kappa\frac{\partial^2 V(x,t)}{\partial x^2} - GV(x,t). \tag{1}$$

We make use of the discretization prescription,

$$\frac{\partial V(x,t)}{\partial t} = \frac{V(i,n+1) - V(i,n)}{\triangle t}, \tag{2}$$

$$\frac{\partial^2 V(x,t)}{\partial x^2} = \frac{1}{2(\triangle x)^2}\left[(V(i+1,n+1) - 2V(i,n+1) + V(i-1,n+1)) + ...\right], \tag{3}$$

$$... + (V(i+1,n) - 2V(i,n) + V(i-1,n))] \tag{4}$$

and

$$V(x,t) = \frac{V(i,n+1) + V(i,n)}{2}. \tag{5}$$

Utilizing the discretization, we obtain,

$$\alpha V(i,n+1) + \beta V(i,n) - \gamma\left[V(i+1,n+1) + V(i-1,n+1) + V(i+1,n) + V(i-1,n)\right] = 0. \tag{6}$$

Suppose, i is 1 or 2. That is, we have a two segment axon. This yields the following simultaneous equations where the $i = 1$ case is displayed first, and then the $i = 2$ case:

$$\alpha V(1,n+1) + \beta V(1,n) - \gamma\left[V(2,n+1) + V(0,n+1) + V(2,n) + V(0,n)\right] = 0 \tag{7}$$

and

$$\alpha V(2,n+1)+\beta V(2,n)-\gamma\left[V(3,n+1)+V(1,n+1)+V(3,n)+V(1,n)\right]=0 \tag{8}$$

Next, we denote, $V(0,n)$ as x, $V(0,n+1)$ as y, $V(1,n)$ as z, $V(1,n+1)$ as a, $V(2,n)$ as b and $V(2,n+1)$ as c, $V(3,n)$ as d and $V(3,n+1)$ as e for this truncated problem. We will assume that $V(\cdot,n)$ is the known case and we will find the solution for the $n+1$ case. The base case, $V(\cdot,1)$ is the initial condition, which is also assumed to be given. Thus induction will complete the entire temporal trajectory. The boundary conditions determine $V(0,\cdot)$ and $V(3,\cdot)$. In this case, those are x, y, d and e. The two simultaneous equations become,

$$\alpha a-\gamma c=-\beta z+\gamma y+\gamma b+\gamma x \tag{9}$$

and

$$-\gamma a+\alpha c=-\beta b+\gamma d+\gamma e+\gamma z. \tag{10}$$

As per the sutra, we find their sum and difference to ease out the solution. Thus,

$$(\alpha-\gamma)a+(\alpha-\gamma)c=-\beta(z+b)+\gamma(y+b+x+d+e+z) \tag{11}$$

and

$$(\alpha+\gamma)a-(\alpha+\gamma)c=\beta(b-z)+\gamma(y+b+x-d-e-z). \tag{12}$$

Upon simplification we get,

$$(\alpha-\gamma)(a+c)=RHS_1 \tag{13}$$

and

$$(\alpha+\gamma)(a-c)=RHS_2 \tag{14}$$

These are easily used to get,

$$a=\frac{RHS_1}{2(\alpha-\gamma)}+\frac{RHS_2}{2(\alpha+\gamma)} \tag{15}$$

and hence,

$$c=\frac{RHS_1}{2(\alpha-\gamma)}-\frac{RHS_2}{(\alpha+\gamma)}. \tag{16}$$

With a and c, we obtain $V(1,2)$ and $V(2,2)$. Using these we would obtain $V(1,3)$ and $V(2,3)$ and so on, the entire temporal trajectory for the two segments is determined.

References

[1] Swami Bharati Krishna Tirtha, Vasudeva Sharana Agrawala, et al. *Vedic mathematics*, volume 10. Motilal Banarsidass Publ., 1992.