Towards a Network Coding View of Ephaptic Synchronization

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Abstract

In a data communication network, the nodes can be organized or arranged in various ways. For small networks, there are some typical arrangements of nodes. In this paper we discuss how the standard butterfly network, a network of six nodes, might be implemented in the nervous system by means of current-mediated coupling between axons. The implementation is in the context of the well-known *network coding gains*. Specifically, we show how two current-coupled axons can exchange information in a 'contact-less' manner. Our strategy provides throughput-enhancement, that is, a network coding gain, resulting from the synchronization of action potentials. We therefore move incrementally towards a better understanding of networks of neurons and how information is transmitted by them and processed in them.

1 Introduction

The nervous system serves various purposes in vertebrates and invertebrates, including higher-order ones such as emotional response, planning and creative activity. The motor control function performed by the nervous system is a case in point. For a specific example, one has only to look at the sino-atrial node where neural commands from the higher regions of the brain and spinal cord are translated into motor regulatory signals for the heart muscles. In humans, the nervous system consists of several major nerves such as the cranial nerves, each having several main components. In a typical nerve, the components include several axons, each of which may or may not be divided further into several segmented regions with 'nodes of Ranvier' demarcating off internodal zones.

The importance of each component of a nerve is highlighted by neurodegenerative diseases wherein one or more components fail. In diseases such as multiple sclerosis (MS), depleted myelination leads to exposure of underlying tissue which enhances the leaked communication current between cells. It is not clearly known whether weakening myelination is useful or pernicious, but it certainly has an effect on the action potential conduction velocity. However, the communication current allows the formation of 'coalitions' or networks of axons, which can act collaboratively. The biological basis of this network formation was first proposed by Arvanitaki in the 1940s through the first paper on 'ephaptic' coupling - 'touching,' but not quite. Our goal for this paper is to show that such collaborative action can lead to enhanced throughput in the associated nerve network. We do this by means of network coding. Simple network coding consists of a graphical network in which the nodes do some simple computation. Network coding was introduced in (Li *et al.*, 2003). It is discussed further in Section 2

This paper is organized as follows: in Section 2 we present a review of network coding in the butterfly network, one of the simplest examples. In Section 3 we perform a review of ephaptic coupling that can lead to synchronization of the coupled axons and show how synchronization in the butterfly configuration, under certain conditions, can lead to throughput gain. Finally, we conclude in Section 4.

2 Network Coding and The Butterfly Network

For a review of the fundamentals of network coding please see the excellent foundational paper by Ahlswede et. al. (Ahlswede *et al.*, 2000). In this section we will describe, in an overview, the essence of the butterfly network and how network coding works in it.

The butterfly network, shown in Figure 1, is a simple network of six nodes. On the top we have two input nodes, which lead in to a central node. The central node has a bottom counterpart which leads out to two output nodes. The input nodes also directly connect to the respective output nodes on the bottom. Suppose the input signals are A and B, respectively, on the two input nodes. These get network coded into A+B in the central line and each destination gets this central line. In addition, the bottom right destination gets the top left input and the bottom left destination gets the top right input. Thus each destination, by means of simple subtraction, is able to get both the messages. Ahlswede et. al. also show that throughput is increased by employing network coding (Ahlswede *et al.*, 2000).

3 Ephaptic Coupling and Synchronization

In current-mediated coupling between axons, impulses traveling with different velocities on two neighboring axons tend to adjust their speeds Scott (2002). The speed adjustment is such that the slower one speeds up and the faster one slows down until they both travel at the same velocity. This velocity is intermediate between the two initial velocities. In this section we show that under certain conditions, the temporally coded information carried by the individual axons prior to synchronization, becomes the sum of the random variables post-synchronization. This allows the possibility of network coding in a butterfly-like network.

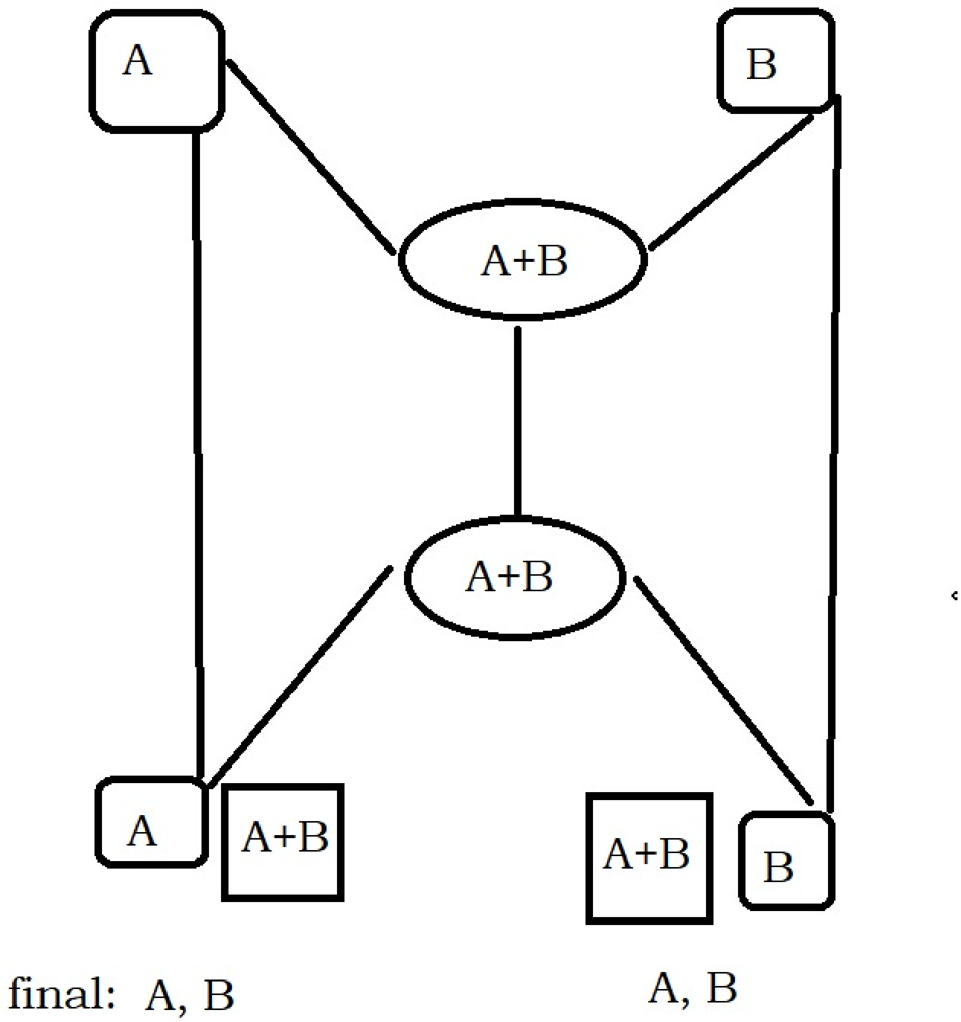

Figure 1: The Butterfly Network

Consider the case of ephaptic synchronization of $N = 2$ axons, A and B. Let v_A and v_B, respectively, be the velocities of the impulses on the two axons. Suppose that the distance to be covered by each impulse is d meters. The time taken by the impulse on axon A is thus $t_A = \frac{d}{v_A}$ seconds and that on axon B is $t_B = \frac{d}{v_B}$ seconds. The velocity due to coupling is a $v_{sync} = \lambda v_A + (1 - \lambda)v_B$. In what follows, we will assume that $\lambda = \frac{1}{2}$. Suppose after synchronization, the two impulses travel together for a distance βd meters where $0 < \beta < 1$. Simple book-keeping then shows that if

$$\beta = \frac{(t_1 + t_2)^2}{t_1 t_2} \tag{1}$$

then

$$t = t_1 + t_2. \tag{2}$$

In other words, the temporally coded 'new' information is the sum of the two input variables. Thus, ephaptic coupling is performing network coding on the two component axonal variables. So any destination that receives the post-coupling temporally coded information, as well as a copy of one of the initial components, is able to compute both of them. In a butterfly-like arrangement therefore, both the destinations get both the components, and all the gains of network coding such as the rise in throughput are available to the nervous system.

4 Discussion and Future Work

Beyond the butterfly network case, our scenario can also be viewed as contact-less information transfer due to ephaptic coupling. In what follows, let A stand for t_1 and B stand for t_2. Say the left axon is from an arbor that does not know B, while the right axon is from an arbor that does not know A. After the left and right axons undergo ephaptic coupling, provided Equation (1) is met, each axon carries A+B. Thus both arbors acquire A+B. Since they also have their original A or B, they effectively have provided each other with their original data without any neurotransmitter exchange.

To summarize, in this paper we have presented a very important observation of contact-less information exchange between axons when they are placed in close proximity of one another. Combined with our earlier observation that ephaptic interactions can be signed (Chawla & Morgera, 2022), we have effectively shown full-fledged information exchange in ephaptic networks. This raises the status of ephaptic networks almost at a par with synaptic ones, even perhaps more efficient since no biophysical molecules need be exchanged. In future work, we would also like to study if there is a trade-off relationship between the network coding throughput gain and the change in the conduction velocity due to demyelinating diseases such as multiple sclerosis; and this could be significant for our understanding of the utility of coupling in these diseases.

References

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