Study of Cross-Information

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Abstract

In this note, the authors define a new quantity called cross-information which is distinct from the mutual information between random variables at the input and output of communication channels.

Introduction In [1] the authors considered:

> a single channel say binary symmetric channel (BSC) with parameter p, BSC(p). Consider another one, BSC(q). Suppose p actually varies with time as per a PDE, and so also q. Then suppose that these two PDEs are coupled. Then we expect to get coupled evolution of the two error exponents.

In the present work we abstract this setting and study it using a new quantity, which we define as follows.

Definition Let X_{in}^1 and Y_{in}^1 be channel inputs when two channels, 1 and 2, are uncoupled. Let X_{out}^1 and Y_{out}^1 be the corresponding outputs. Let (X_{in}, Y_{in}) be the channel inputs to the coupled version of the two channels 1 and 2, and let (X_{out}, Y_{out}) be the corresponding output. We say,

$$I_c(input; output) \equiv I((X_{out}, Y_{out}); (X_{in}, Y_{in})) - I(X_{in}^1; X_{out}^1) - I(Y_{in}^1; Y_{out}^1). \quad (1)$$

Along the lines of the analysis in [2, 3], we investigate the concavity property of this definition.

Concavity of Cross-Information We find that the cross-information is concave in the uncoupled channels for a fixed coupling. So one can look for optimal (maximal) marginal or uncoupled channels.

Convexity of Cross-Information We find that the cross-information is convex in the coupling for fixed marginal channels, consistent with the results obtained in Figure 1 of [4].

Limitation: Entropy Amplification Applied In [5] we showed that

$$H(X_{out}) \geq \max \left\{ H(X^1_{out}), H(Y^1_{out}) \right\} \tag{2}$$

and

$$H(Y_{out}) \geq \max \left\{ H(X^1_{out}), H(Y^1_{out}) \right\}. \tag{3}$$

since upon synchronization the time-coded random variables delivered at the output of both (coupled) axons are identical. This result is intertwined with the cross-information as defined in this work and in future work, the relationship should be closely examined.

Conclusion In this note we examined ephaptically coupled channels by introducing a "cross-information." In a previous paper [4] we had studied their mutual information (the coupled case) directly using simulations.

References

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