Sphere-packing in Spherical Spaces

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April 30, 2024

Abstract

In this very brief note, the authors propose that instead of Euclidean R^3 as the background geometry for sphere packing as it is used in the derivation of the gaussian channel capacity, a spherical geometry be considered.

In [1], Einstein writes:

> From the latest results of the theory of relativity it is probable that our three-dimensional space is also approximately spherical, that is, that the laws of disposition of rigid bodies in it are not given by Euclidean geometry, but approximately by spherical geometry, if only we consider parts of space which are sufficiently great.

He then goes on to explain spherical geometry in a very enthralling way, and also briefly touches upon elliptical geometry.

If our actual ambient space is non-Euclidean it might be incorrect to ignore this reality. In the present note we raise the question if the sphere packing construction which is used in determining the capacity of the additive white Gaussian noise channel [2] can be looked at in a different background than Euclidean R^3 space.

If we replace Euclidean geometry with spherical geometry in sphere packing, there may be some connections drawn with the packing of stellar light cones in the universe, and therefore the "capacity" of the night sky. Olbers' paradox [3] might also be re-examined in this new light. Further, there may be some thing to say related to gravitational lensing [4]. These considerations would be interesting to examine in future work.

References

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[2] Thomas M. Cover and Joy A. Thomas. *Elements of Information Theory*. 2005.

[3] George Smoot and Keay Davidson. *Wrinkles in time*. William Morrow and Company, 1993.

[4] Kip S Thorne, Charles W Misner, and John Archibald Wheeler. *Gravitation*. Freeman, 2000.