Some Progress on Axonal Quantum Computation in the Markin Approximation

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Abstract

In this note, the authors consider Markin's approximation to the ionic current, in the context of the Frankenhaeuser-Huxley equations. Using a Green's function approach, they derive the solution to the equations. This is a first step in a program to determine the exact quantum computation sequence in axon tracts.

It is well recognized that the Hodgkin Huxley equations or the corresponding Frankenhaeuser-Huxley equations are difficult to solve exactly due to the complexity of the ionic current term [1]. Markin and co-authors circumvent this difficulty by assuming a general form of the ionic current [2, 3].

In a paper by Markin and Chizmadzhev cited within [1], they present a piecewise-linear approach to the ion channel current. In later work, Markin uses this formulation to solve the single and multiple axon coupling situation. In Section 1, we set the problem up and then use the Green's method for solving the two-variable second order partial differential equation that arises. We conclude in Section 2.

1 Problem Setup and Solution

We utilize the Markin model for the ion currents. They can be succinctly written using Heaviside step functions. In particular, let $\mathcal{I}(z,t)$ stand for the ionic current at a time t at a node. We can write,

$$\mathcal{I}(z,t) = s_1(t) \cdot s_2(t) \tag{1}$$

where

$$s_1(t) = j_1 + (j_1 + j_2) \cdot \Theta(t - \tau_1) \tag{2}$$

and

$$s_2(t) = \Theta(t) - \Theta(t - \tau_2) \tag{3}$$

and where j_1, j_2, τ_1, τ_2 are the parameters in Markin's model and $\Theta(\cdot)$ is the Heaviside step function. Next recall that in the no-coupling case, the relevant PDE can be written in terms of a differential operator $\mathcal{L}$ defined as,

$$\mathcal{L} = C\frac{\partial}{\partial t} + \kappa\frac{\partial^2}{\partial z^2} + G \tag{4}$$

where C is the membrane capacitance and G is the myelin conductance and κ is a constant. The PDE, expressed in these terms is,

$$\mathcal{L}V(z,t) = \mathcal{I}(z,t). \tag{5}$$

In the Green's approach we replace the forcing term with Dirac deltas and look for the corresponding solution which is known as the Green's function $\tilde{G}((z,t),(\xi,\xi_t))$. Thus the equation we want to solve becomes,

$$\mathcal{L}\tilde{G}(z,t) = \delta(z-\xi)\cdot\delta(t-\xi_t). \tag{6}$$

Using an LLM, we find,

$$\tilde{G}((z,t),(\xi,\xi_t)) = \frac{1}{2C}\sqrt{\frac{\pi C}{\kappa|z-\xi|}}\cdot e^{\frac{G|z-\xi|}{C}}\cdot e^{-\frac{(t-\xi_t)^2 C}{4\kappa|z-\xi|}}. \tag{7}$$

Thus, the transmembrane voltage solution can be written as,

$$V(z,t) = \iint \tilde{G}((z,t),(\xi,\xi_t))s_1(\xi_t)s_2(\xi_t)d\xi d\xi_t + b.i. \tag{8}$$

where $b.i.$ stands for boundary integral.

2 Discussion

If the above solution is verified manually, the next steps are straightforward. We will extend this approach to three ephaptically coupled axons [4], and find the explicit expression for $V_i(z,t)$ with $i=1,2,3$, also forming $\tilde{V}_a(z,t) = V_2(z,t) - V_1(z,t)$ and $\tilde{V}_b(z,t) = V_3(z,t) - V_1(z,t)$. Next, we will extend the approach to the case when a gravitational wave (indicated by the strain h) is impinging on the tract of three axons and find, analytically, $\tilde{V}_{b,h}(z,t) - \tilde{V}_b(z,t)$. This will be the perturbation potential that will enter into the environment term for controlling the quantum harmonic oscillator of [5]. This program was outlined in detail in the Zenodo preprint [6].

References

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