Shannon Approach to Bosons

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Abstract

In this note the authors consider the indistinguishability property of bosons and evaluate how it can be incorporated into a discussion of communicating messages using bosons.

Dedication: This work is dedicated to the kriya yoga master Paramahamsa Hariharananda Giri and Prof. Sanjoy Kumar Mitter.

Consider two cases.

1. Case A. We transmit the "number" n_{E_i} of particles possessing various energies in the set $E_1, E_2, ...$ The order of transmission is not important, but all i must be sent. There are k such i.

2. Case B. We transmit the particles themselves, each possessing an energy in the set $E_1, E_2, ...$ All the n particles must be sent.

Consider the following examples of the two cases.

1. Case A Example. $n_{E1} = 2, n_{E2} = 5, n_{E3} = 6, ...$ Messages, $m_1 \equiv n_{E3}, m_2 \equiv n_{E1}, m_3 \equiv n_{E3}, ...$

2. Case B Example. Messages, $m_1 \equiv particle - 5, m_2 \equiv particle - 6, m_3 \equiv particle - 1, m_4 \equiv particle -$

From Case B we can recover Case A over a time period n of output observation, except for the order, but since the names of the particles are unknown, from Case A we cannot recover Case B. When the particles are indistinguishable, Case A and Case B, after their full execution, are identical.

Next, suppose that

$$X_A^{(k)} \to X_B^{(n)} \to Z \tag{1}$$

is a Markov chain, where the subscripts represent the cases, and Z is the estimate of $X_A^{(k)}$. Here $X_A^{(k)}$ is the random variable which represents the full sequence of messages in Case A, within order permutations, and $X_B^{(n)}$ is the random variable which represents the full sequence of messages in Case B; it represents the complete transmission sequence of length n. Then, by the data processing inequality,

$$I_{MaxwellBoltzmann}(X_A^{(k)}; Z) \leq I(X_A^{(k)}; X_B^{(n)}). \tag{2}$$

As n increases, the RHS will converge to $H(X_A^{(k)})$.

$$I(X_A^{(k)}; Z) \leq H(X_A^{(k)}). \tag{3}$$

Further, if the particles are indistinguishable then Equation (1) does not hold as presented. So we have the condition that

$$I_{Boson}(X_B^{(N)}; Z) \leq H(X_B^{(N)}). \tag{4}$$

Next consider the relation between $H(X_A^{(k)})$ and $H(X_B^{(N)})$. Clearly, in the distinguishable case, the latter is larger. We have,

$$I_{MaxwellBoltzmann}(X_A^{(k)}; Z) \leq H(X_A^{(k)}) \leq H(X_B^{(N)}). \tag{5}$$

And in the indistinguishable case, both are identical. We have,

$$I_{Boson}(X_B^{(N)}; Z) \leq H(X_B^{(N)}) = H(X_A^{(k)}). \tag{6}$$

Thus we have obtained certain distinctions between the Bosonic and Maxwell-Boltzmann cases. It remains to be seen if these can be exploited in data compression, feedback and transmission. Thus in future work, we would like to consider the relation of the above distinctions to the work in [1]. Furthermore, Case A is the transmission of a p.m.f. and Case B is also the transmission of a p.m.f., but for bosons. Thus, we can ask about the relation of the above to [2] and similar works.

References

[1] Samuel L Braunstein and Carlton M Caves. Information-theoretic bell inequalities. *Physical review letters*, 61(6):662, 1988.

[2] Travis Gagie. Compressing probability distributions. *arXiv preprint cs/0506016*, 2005.