Quantum Anytime Information Theory: A Proposal

1. Introduction

Quantum technologies are entering slowly into the mainstream of society. The entire field of Electrical Engineering and Computer Science can in some way be applied to quantum mechanics, in order to generate quantum engineering. In this proposal, we explore one aspect of quantum engineering, namely quantum information flow in quantum channels when used for quantum feedback control.

This proposal exploits the new NISQ quantum computing technologies [13] which are fast becoming widely and easily available to the common researcher in order to push forward the envelope of our understanding of quantum control and information. The key message of this proposal is that we are at a juncture in time when it is feasible for the first time to carry out some of the experiments to validate our theoretical framework of quantum control and information. This message will be conveyed by demonstrating a very simple problem, that of tracking a quantum source over a quantum channel.

This proposal is structured as follows. In Section 2, we survey the literature on the interaction between information and control in the quantum context. In Section 3, we present the background. In particular we show some rudimentary example code implemented on the IBM quantum computing platform. Additionally, we present our research proposal. Finally, we conclude in Section 4 with suggestions for future work.

2. Literature Review

In the paper by Mabuchi and Hamerly [1], the authors discuss coherent feedback in the quantum setting. This is opposed to classical feedback which has also been studied extensively in the quantum case. Whereas classical feedback involves a measurement of the forward output, coherent feedback avoids this measurement step. In this paper, the authors present the results of quantum control experiments that implement their novel theory of coherent feedback control. They do not invoke quantum simulations.

In the paper by James and Gough [2], the authors present a new formalism for the study of open quantum systems. They then apply this theory to a variety of different settings. The formalism is based on K. R. Parthasarathy's quantum stochastic calculus [3] and as such is very versatile. The work however is entirely theoretical, dating to before the advent of commercial quantum simulation systems such as the IBM setup.

In the paper by Khaneja, Brockett and Glaser [4], the authors look at the NMR problem of regulating the state-dynamics of spin systems using controlling pulses that are well-designed. The paper uses much advanced mathematics such as Lie algebras, in order to show the optimality of the proposed method of spin state transfer. The study is entirely theoretical.

In the paper by Belavkin [5], we also find a presentation of quantum stochastic calculus and quantum nonlinear filtering. Belavkin cites [3] and treats especially the case of open quantum systems undergoing nondemolition measurements. This theoretical work however does not look at simulations using quantum computers [6].

[7] discusses the quantum noisy channel coding theorem, which is important from the perspective of looking at the interaction between quantum information and quantum control. The paper does not look at this interaction at all; it is a work in quantum Shannon theory. In the same vein, [8] looks at quantum continuous measurements, which are important for developing this intersection between control and information in the quantum context. The measurement problem may even preclude a development of an anytime quantum information theory [12]. This needs to be investigated in detail.

In the paper by Patel and Tiwari [13], the authors propose a modification to the current set up for state discrimination using the IBM quantum computing system. Their proposed modification performs measurably better than the current one. This paper is the first step in our study of quantum control and information as state discrimination is a fundamental task before and during information processing and utilization.

In the context of the literature surveyed above, our project is novel in seeking to establish a quantum version of anytime information theory. When specialized to the coherent case, where measurements of quantum systems which collapse the system states are bypassed, this theory will tell us the fundamental (governed by physical law) limits of stabilizing quantum information transfer. For example, what are the feasible rates when I wish to stabilize a given plant? With this, the designer of a particular quantum control system will know whether he/she is operating their system in an optimal way or not, and also possibly give some insight on how to improve the operation.

The quantum anytime codes so designed, will aid in synchronization of the communication between two spatially separated parties. As a consequence of this, the new class of codes will be useful when “real-time” applications of quantum computation are being designed. If we envision a future in which quantum data is freely and widely used, just as the classical internet is presently being used, then real-time applications will be booming. By conducting this study at this early stage in the quantum-era, we are positioning ourselves ahead of the game by about

20-50 years.

3. Background and Proposal

3.1 Control and Information

A basic (classical) feedback control system is shown in Figure 1. Initial work on a quantum communication-only version of this system has been completed [18] and is presently being extended to the case of noise and entanglement. In relation to the present proposal, we will utilize the presentation of Astrom and Murray [9] for output feedback control theory and Cover and Thomas [10] for information theory. Further, we will need the setups of [15], [16] and [17] which are essential for our investigation of joint transmission of quantum information and energy, for purposes of quantum feedback control. For quantum control and quantum information we will utilize the reference [11] which contains several papers that begin to lay the foundations of interaction between quantum control and quantum information.

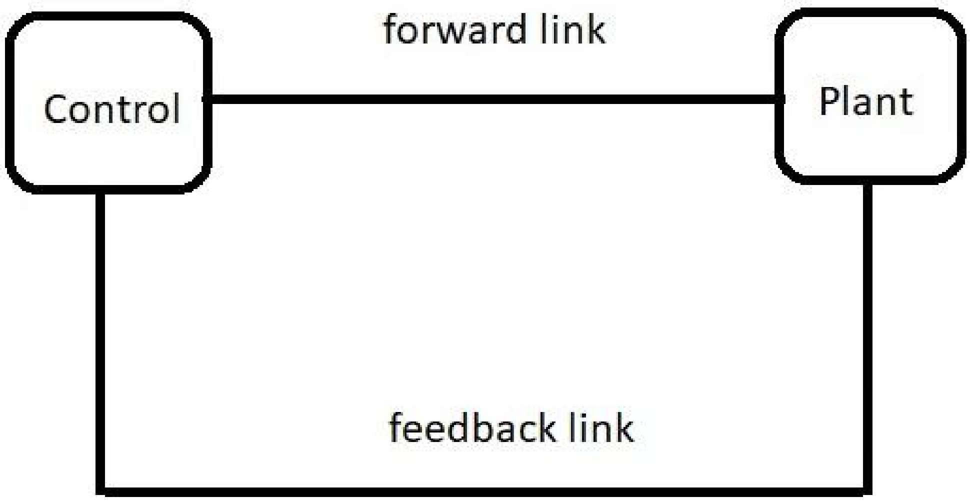

Figure 1: Basic schematic of feedback control. On the left is the controller and on the right is the plant. The arrows represent the flow of information. The forward and feedback links may be considered capable of simultaneously transmitting information as well as energy, in the quantum setting.

3.2 Simulation

We will utilize IBM's quantum lab in order to run simulations of quantum circuits. As an example, for illustrative purposes, here we show the code for simulating a single qubit, undergoing rotation followed by measurement.

```
import qiskit # call the qiskit's module
from math import pi

qr = qiskit.QuantumRegister(1) # call a quantum bit (or qubit)
cr = qiskit.ClassicalRegister(1)
```

```
qc = qiskit.QuantumCircuit(qr,cr)
qc.u(pi/2,pi/2,pi/2,qr)
qc.measure(qr,cr)
qc.draw()
```

The output of the above lines is shown below:

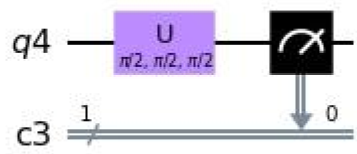

Figure 2: A circuit where the quantum bit is rotated and then measured. The measurement result is stored in a classical register.

In order for us to execute the above circuit on the IBM simulator, we need the following line of code:

```
job = qiskit.execute( qc, qiskit.BasicAer.get_backend('qasm_simulator') )
```

The result obtained is as follows:

```
print( job.result().get_counts() )
{'1': 538, '0': 486}
```

It can be plotted as a histogram using:

```
qiskit.visualization.plot_histogram(job.result().get_counts(qc))
```

This results in the histogram shown in Figure 3. Next we run the same job on an actual quantum system. The results (probabilities for zero and one output) are shown in Figures 4 and 5. As can be noted, the values of the probabilities are slightly different from the simulator results. The code used is provided below:

```
qiskit.IBMQ.load_account() # load the token
provider = qiskit.IBMQ.get_provider('ibm-q-research-2') # select the provider
backend = provider.get_backend('ibm_perth') # select the name of the quantum computer to use
print("real device:",backend.name())
job = qiskit.execute( qc, backend )
print( job.result().get_counts() ) # show the result
```

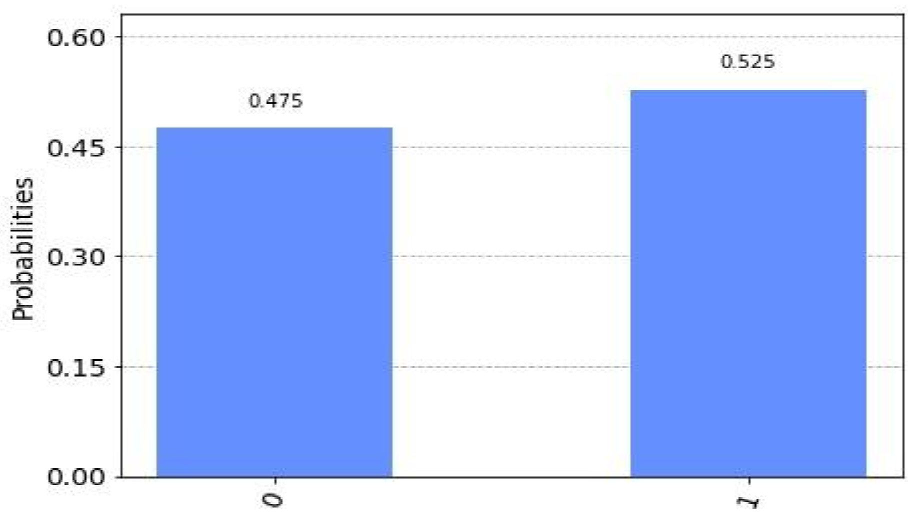

Figure 3: Qiskit simulator output of the quantum circuit shown in Figure 2.

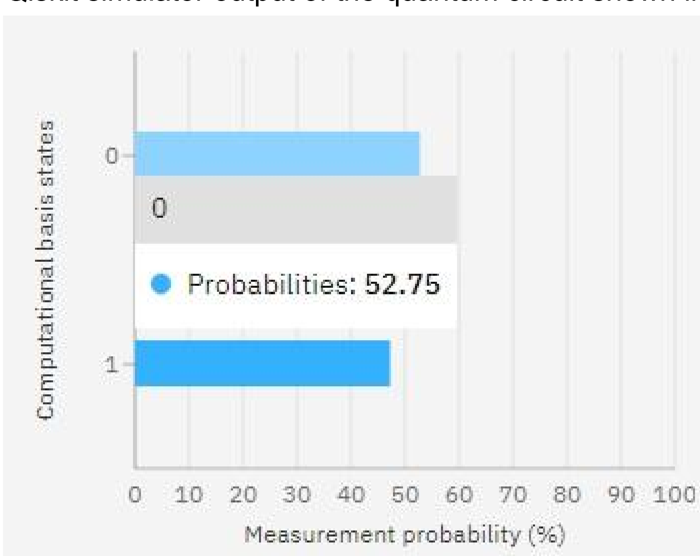

Figure 4: Qiskit (actual, quantum) output of the circuit shown in Figure 2, specifically, the probability of ‘0’.

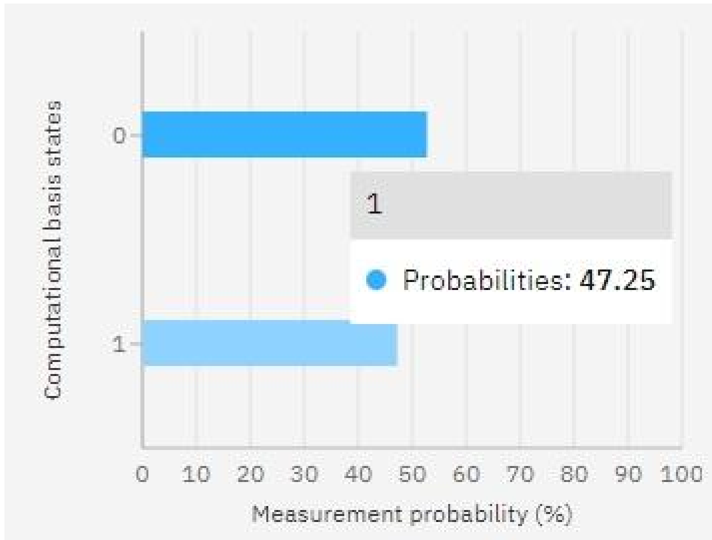

Figure 5: Qiskit (actual, quantum) output of the circuit shown in Figure 2, specifically, the probability of ‘1’.

3.3 The Proposal

First Part Based on Theory and Simulations

In this section we discuss our specific proposal, under which the setup that we show in Figure 1 will be simulated in Qiskit and its properties examined, towards the development of an ‘anytime’ quantum information theory[3]. The theoretical development will be carried out in parallel with the experimental and simulation studies, and all these three will inform each other, for a consistent outcome. Many of the rudimentary components of such a theory are possibly available in disparate papers such as those of Mabuchi [1], James and Gough [2], and they need to be assimilated into a coherent framework.

In [17], the anytime framework of [16] is utilized as a feedback code. In [19], a similar study was done for a Poisson channel, with initial work towards the analysis of a Poisson anytime code. This gives us the background to work towards the full quantum case, initiated in [18]. In [20], we looked again at the continuous Gaussian feedback channel as a communication system, but this time with perfect feedback. This illuminated for us the need for subtle quantum measure theoretic [21] notions to answer the full quantum question.

[3] Recall that anytime information theory [16] in the classical setting investigates the capacity of a classical channel when it is to be used for communicating the control variables in a feedback control setting.

The **key contribution** of this proposal will be to answer the question of whether we need a notion of *anytime* quantum capacity. It is the measurement question which complicates the issue here; otherwise there would be no trouble in following exactly the classical development.

Second Part Based on Experiment

We propose herein to verify our simulational and theoretical results by actual laboratory measurements. Towards this end, we propose to follow the setup in [22, 23]. At a later stage, we will have the know-how to modify this setup to do a full-fledged streaming quantum anytime information flow study.

3.4 Environmental Impact

It has been seen that in the classical domain, the internet has solved many communication and computation related problems for man, while having a minimal environmental footprint. In a similar way, we can envision that when quantum data is being used for quantum control, the environmental impact is minimal. Since the theory and simulations developed will be generic and not targeted to specific atoms or control applications, we may safely say that there won't be a negative impact of the developed techniques in the first part of this proposal.

In the second part where we are using specific technologies such as lasers and optical parametric oscillators, there will be safety issues for human users that approach the laboratory. Apart from these issues, environmental impact will be minimal.

4. Conclusion

In this document we have presented a proposal to simulate and study quantum feedback control over communication channels utilizing IBM's Qiskit quantum platform and an experimental setup [22, 23]. By means of figures it was illustrated that there are differences between a quantum simulation and running an actual quantum experiment. Theory is to be developed utilizing known principles in combination with simulation and actual quantum experiments. A timeline for the completion of our project has been provided at the end of this document. It is hoped that this work will push forward the state of the art in our understanding of quantum mechanics as well its engineering applications. If we are successful in the simulation aspect, we will be able to validate the theory using the full quantum coherent feedback control system [22].

References

1. Hamerly, R., & Mabuchi, H. (2012). Advantages of coherent feedback for cooling quantum oscillators. Physical review letters, 109(17), 173602.
2. James, M. R., & Gough, J. (2008, December). Quantum behaviors and networks. In 2008 47th IEEE Conference on Decision and Control (pp. 4552-4557). IEEE.
3. KR Parthasarathy. (1992). An introduction to quantum stochastic calculus (Vol. 85). Springer Science & Business Media.
4. Khaneja, N., Brockett, R., & Glaser, S. J. (2001). Time optimal control in spin systems. Physical Review A, 63(3), 032308.
5. Belavkin, V. P. (1992). Quantum stochastic calculus and quantum nonlinear filtering. Journal of Multivariate analysis, 42(2), 171-201.
6. García-Pérez, G., Rossi, M. A., & Maniscalco, S. (2020). IBM Q Experience as a versatile experimental testbed for simulating open quantum systems. npj Quantum Information, 6(1), 1-10.
7. Klesse, R. (2008). A random coding based proof for the quantum coding theorem. Open Systems & Information Dynamics, 15(01), 21-45.
8. Jacobs, K., & Steck, D. A. (2006). A straightforward introduction to continuous quantum measurement. Contemporary Physics, 47(5), 279-303.
9. Åström, K. J., & Murray, R. M. (2010). Feedback systems. Princeton university press.
10. Cover, T. M. (1999). Elements of information theory. John Wiley & Sons.
11. Belavkin, V. P., & Guta, M. (Eds.). (2008). Quantum stochastics and information: statistics, filtering and control. World Scientific.
12. Anant Sahai. *Personal Communication*.

13. Preskill, J. (2018). Quantum computing in the NISQ era and beyond. Quantum, 2, 79.
14. Patel, T., & Tiwari, D. (2020, November). Disq: a novel quantum output state classification method on ibm quantum computers using openPulse. In Proceedings of the 39th International Conference on Computer-Aided Design (pp. 1-9).
15. Varshney, L. R. (2008, July). Transporting information and energy simultaneously. In 2008 IEEE international symposium on information theory (pp. 1612-1616). IEEE.
16. Sahai, A., & Mitter, S. (2006). The necessity and sufficiency of anytime capacity for stabilization of a linear system over a noisy communication link—Part I: Scalar systems. IEEE transactions on Information Theory, 52(8), 3369-3395.
17. Chawla, A. (2006). Reliability of a Gaussian channel in the presence of Gaussian feedback (SM dissertation, Massachusetts Institute of Technology).
18. Chawla A (2012). Reliability of a classical-quantum communication system with noisy feedback. QIP 2012. Poster.
19. Chawla A, Amouri A and Morgera S D (2012). On the reliability function of a neuronal channel when used with neuronal feedback. Award winning poster at USF Engineering Research Day 2012.
20. Chawla A and Morgera S D (2021). On the continuous time additive Gaussian noise channel in the presence of perfect feedback. Information Theory Workshop 2020. pp. 1-5.
21. Hamhalter, J. (2013). Quantum measure theory (Vol. 134). Springer Science & Business Media.
22. Crisafulli, O., Tezak, N., Soh, D. B., Armen, M. A., & Mabuchi, H. (2013). Squeezed light in an optical parametric oscillator network with coherent feedback quantum control. Optics Express, 21(15), 18371-18386.
23. Singh, S. P., Mondal, S., Hussain, K., & Datta, P. K. (2011). Development of Optical Parametric Oscillator Tunable in the Range 970-1460 nm. Defence Science Journal, 61(4), 377.

		Name	Duration	Tentative Start	Tentative Finish	Predecessors	Resource Names
1		Simulate a Quantum Channel on Qiskit	60 days	4/15/22 8:00 ...	7/7/22 5:00 PM		Aman Chawla[20%]
2		Simulate a Quantum System to be Used as a Plant	60 days	6/15/22 8:00 ...	9/6/22 5:00 PM		Aman Chawla[20%]
3		Simulate a Quantum System to be Used as a Controller	60 days	8/15/22 8:00 ...	11/4/22 5:00 PM		
4		Simulate Figure 1	60 days	10/15/22 8:0...	1/6/23 5:00 PM		
5		Write-up Results	90 days	2/15/23 8:00 ...	6/20/23 5:00 PM		
6		Simulate an Unstable Quantum System	120 days	8/15/23 8:00 ...	1/29/24 5:00 PM		
7		Simulate Figure 1 with an Unstable Plant	150 days	12/15/23 8:0...	7/11/24 5:00 PM		
8		Simulate Source Tracking	120 days	5/15/24 8:00 ...	10/29/24 5:00 ...		
9		Normal Probabilistic Analysis	120 days	8/15/24 8:00 ...	1/29/25 5:00 PM		
10		Unstable Probabilistic Analysis	60 days	12/15/24 8:0...	3/7/25 5:00 PM		
11		Anytime Capacity	60 days	2/15/25 8:00 ...	5/9/25 5:00 PM		
12		Write-up Results Final	30 days	4/15/25 8:00 ...	5/26/25 5:00 PM		