Photon Mean Free Path in Turbulence: First Investigation

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Abstract

In this note the authors consider free space optical communications. The photons involved must traverse eddies of varying refractive indices and sizes. The authors investigate the mean free path of a photon traversing these eddies. This preliminary study indicates a quintic-root dependence of the mean free path on eddy size.

1 Introduction

In free-space optical (FSO) communications, the Earth's atmosphere introduces refractive index fluctuations due to turbulence. These fluctuations act over a wide range of spatial scales, from millimeter-sized inner-scale eddies (l_0) to hundred-meter outer-scale eddies (L_0), and affect photon propagation by altering the phase and amplitude of the optical wavefront.

This note is structured as follows. Section 2 investigates the mean free path of photons traversing eddies. Sections 3 and 4 look at eddy sizes and coherence respectively. Section 5 provides a numerical illustration of the mean free path relationship introduced in Section 2. Finally, we conclude in Section 6.

2 Mean Free Path of a Photon

Photons traveling through turbulence experience phase shifts from refractive index inhomogeneities. The effective mean free path λ_{eff} can be modeled as a function of eddy size using a random walk interpretation [1]:

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$$\lambda_{\text{eff}}(l) \propto l^{1/5}, \quad \text{for } l_0 \ll l \ll L_0$$

This scaling arises from integrating the Kolmogorov turbulence spectrum over eddy sizes and applying a sensitivity cutoff based on the optical wavelength.

3 Fast vs. Slow Fading Regimes

The size of turbulent eddies determines the fading regime:

- **Small eddies** ($l \sim l_0$): cause *fast fading* via scintillation and speckle effects, with rapid temporal fluctuations. This is not normally incorporated in FSO channel models[1].

- **Large eddies** ($l \sim L_0$): cause *slow fading*, such as beam wander and defocus, over long spatial and temporal scales.

4 Spatial and Temporal Coherence

Let G be the spatial coherence length (mean free path) in the slow-fading regime. The corresponding coherence time T_G is:

$$T_G = \frac{G}{v}$$

where v is the transverse wind velocity. The pair (G, T_G) defines the spatiotemporal coherence window in which the channel can be considered quasi-static.

5 Numerical Illustration

Figure 1 illustrates the dependence of λ_{eff} on eddy size l.

6 Summary

Understanding the interplay between eddy size and fading behavior is critical for the design of adaptive FSO systems. The spatial coherence length G and coherence time T_G provide useful parameters for estimating how frequently the channel must be re-estimated or compensated.

[1] Prof. Abhishek Dixit, IIT Delhi, personal communication.

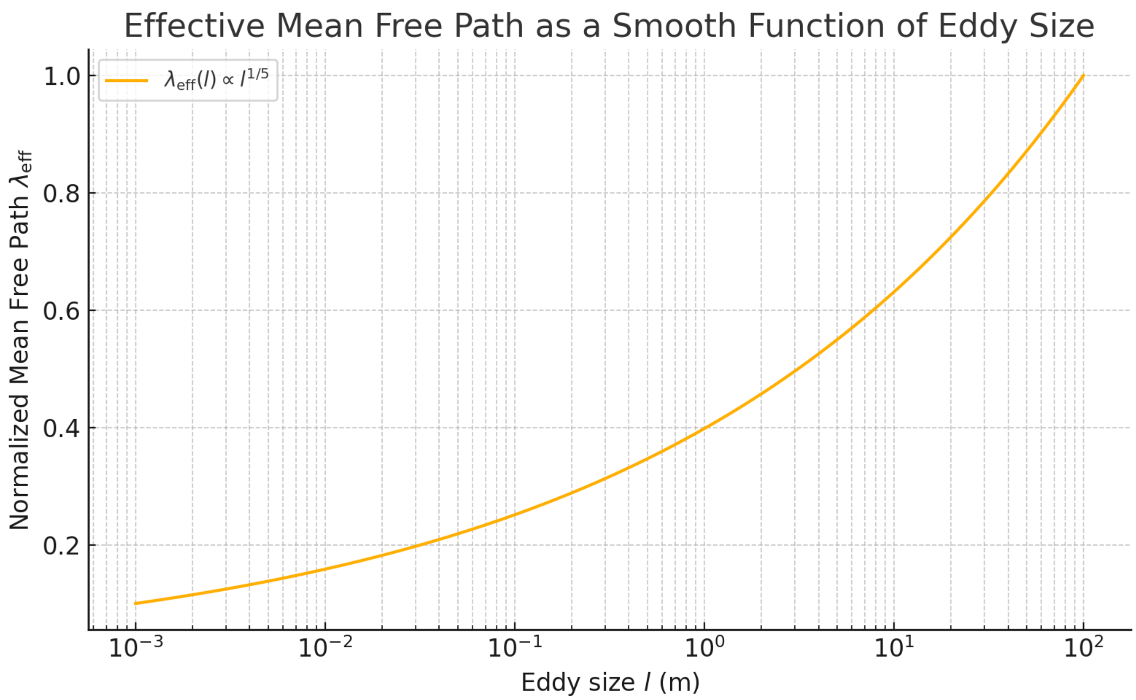

Figure 1: Mean free path as a function of eddy size.

References

[1] R Mitalas and KR Sills. On the photon diffusion time scale for the sun. *Astrophysical Journal, Part 1 (ISSN 0004-637X), vol. 401, no. 2, p. 759, 760.*, 401:759, 1992.