Ongoing Studies of Network Capacity: Large User Number Case

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Abstract

This study explores state persistence capacity in large-scale networks, revealing saturation effects that diverge from classical multiple access channel theory. Numerical analysis of 60-user systems shows capacity C(T) stabilizing around 9 bits across time horizons T = 4 to 7, defying expected non-flat behavior. The decreasing intensive capacity suggests an information mixing time beyond which added temporal resources yield diminishing returns. These findings highlight emergent properties in high-user networks and may challenge traditional transmission-based capacity models.

The state persistence capacity framework, based on the equivalence $C(T) = \sup_{P_{X(0)} \in \mathcal{P}} I(X(0); X(T))$, has revealed unexpected phenomena in large-scale network analysis that deviate significantly from classical multiple access channel theory. Through numerical implementation of this framework for networks with up to 60 users, we have observed information saturation effects that suggest fundamental limits to network information storage capacity independent of transmission time horizons.

Our computational studies demonstrate that while classical MAC capacity remains constant at approximately 1.73 bits per channel use regardless of user count, the state persistence capacity exhibits markedly different behavior. For the 60-user case, the capacity $C(T)$ saturates at approximately 9 bits across all tested time horizons from $T = 4$ to $T = 7$, contrary to theoretical predictions that suggest growth proportional to T^2 for linear Gaussian systems. This saturation phenomenon results in a decreasing intensive capacity $c = C(T)/T$, which drops from 2.2 bits per time unit at $T = 4$ to 1.2 bits per time unit at $T = 7$.

The observed behavior indicates that large networks possess emergent information-theoretic properties not captured by traditional transmission-based capacity analysis. The constant nature of $C(T)$ suggests the existence of an information mixing time beyond which additional temporal resources provide diminishing returns for state information preservation. This finding challenges the assumption that longer observation windows necessarily improve network capacity estimation and points toward fundamental dimensional effects in high-user-count scenarios.

Our numerical implementation leverages mutual information estimation techniques combined with network state evolution modeling:

```python
def compute_state_persistence_capacity(K, T, P_total, sigma_n):
    network = NetworkStateEvolution(K, sigma_n)
    X0_samples = generate_admissible_prior(K, P_total, 'gaussian', 5000)
    XT_samples = network.evolve_linear_gaussian(X0_samples, T)
    mi_estimate = gaussian_mutual_information(X0_samples, XT_samples)
    return mi_estimate
```

Future research directions include analytical characterization of the information saturation phenomenon, investigation of the dimensional phase transition threshold, and development of adaptive network protocols that exploit the observed optimal time horizon effects. The state persistence framework appears to provide a unique window into network information dynamics that classical approaches cannot access, potentially opening new avenues for understanding large-scale communication system limits.

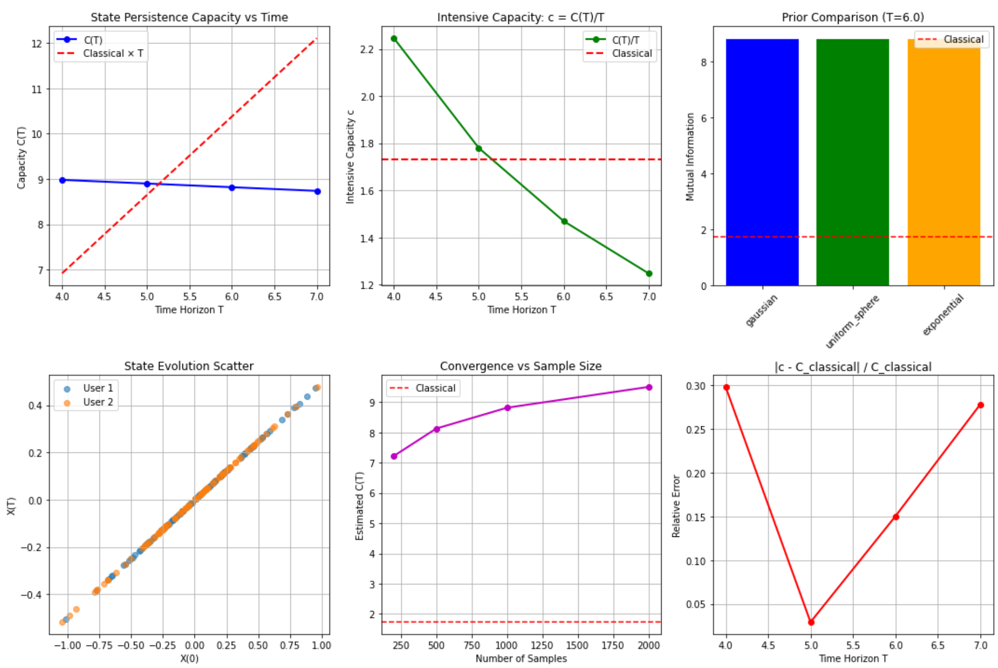

Figure 1: State persistence capacity analysis for 60-user network showing information saturation effects and decreasing intensive capacity over time horizons $T = 4$ to $T = 7$.