On the Sinusoidal Perceptron in a Different Form

Aman Chawla

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Abstract

For a sinusoidal activation function and small arguments, the authors propose in this note that the perceptron weight update equation can be rewritten as a set of coupled ordinary differential equations, where the coupling is proportional in strength to the input data.

In this note, the authors derive (derivation follows) that the weight update equation of a Rosenblatt perceptron for a small-argument sinusoidal activation function is given by,

$$\frac{dw_i}{dt} = \eta x_i^2 w_i + \left(\sum_{k \neq i} \eta x_k w_k \right) x_i + (\eta b N - d) x_i \tag{1}$$

where w_i are the weights, η is the learning rate parameter, x_i are the data points, b is the bias, N is the total number of data points and d is the desired output.

To derive Equation (1), we start with the traditional weight update equation for a perceptron,

$$w(n+1) = w(n) + \eta(y(n) - d(n)) * x(n). \tag{2}$$

Collecting the weight containing terms on the LHS and expressing the unit time step as $\triangle t$, we get,

$$\frac{w(n + \triangle t) - w(n)}{\triangle t} = \eta * (y(n) - d(n)) * x(n). \tag{3}$$

This can be used to obtain the differential equation,

$$\frac{dw}{dt}|_{(n+1)\triangle t} = \eta * (y(n\triangle t) - d(n\triangle t)) * x(n\triangle t). \tag{4}$$

Next we note that the output is nothing but the weighted inputs, summed, and passed through the activation function. Thus,

$$\frac{dw_i(t)}{dt} = \eta * [\phi(\sum_{i=1}^{N} w_i(t) * x_i(t) + b_i) - d(t)] * x_i(t). \tag{5}$$

Assume a sinusoidal activation,

$$\phi(\cdot) = sin(\cdot), \tag{6}$$

noting that

$$sin(x) \sim x \tag{7}$$

for small x. Assuming that all the biases are identical, and after some regrouping, this yields Equation (1) which allows us to think of the Rosenblatt perceptron as a collection of dynamical weight equations, which are mutually coupled by the data that is input into the perceptron. The analogy here is with the ephaptic coupling equation derived in [1, 2]. In future work, we intend to pursue this analogy further and examine the consequences[1].

References

[1] Aman Chawla, Salvatore Morgera, and Arthur Snider. On axon interaction and its role in neurological networks. *IEEE/ACM Transactions on Computational Biology and Bioinformatics*, 2019.

[2] Aman Chawla. *On Axon-Axon Interaction via Currents and Fields.* PhD thesis, University of South Florida, 2017.

[1] Since most entities in Equation (1) are random variables, unlike in the ephaptic coupling equation, we can consider their corresponding mean value processes. If we take the expected value of Equation (1), we obtain

$$\frac{dE[w_i]}{dt} = \eta E[x_i^2 w_i] + \sum_{k \neq i} \eta E[x_k w_k x_i] + (\eta b N - d) E[x_i] \tag{8}$$

While $E[x_i]$ is the mean of the i-th data point, $E[x_i^2 w_i]$ can be thought of as its weighted variance and $E[x_k w_k x_i]$ as the weighted cross-covariance of the k-th data point with the i-th one. It is notable that the second term on the RHS is the discounted (by the learning rate parameter η) sum over $N-1$ of the weighted cross-covariances, with k different from i. Thus the k-th equation affects the i-th one only via the weighted cross-covariance.