On the Irrelevance of Angular Geometry for Very Large Tracts

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Abstract

In this note, the authors show that as the size of an axon tract goes to infinity (number of axons tending to be very large), the distribution of angular inclinations approaches a Gaussian curve and the universal coordinate becomes inclination-independent. This may be why large-scale brain recordings do not show any impact of metric perturbations or any role for axonal angular inclinations. Thus our theory appears to converge to the correct classical limit in the case of very large sized axon tracts.

In [1] a theory has been put forward which shows a significant role for tract geometry. In particular, the inclinations of the axons to the tract axis have been shown to affect action potential propagation profiles. This theory was further extended to show the impact of metric perturbations on these profiles [2]. In [3] a theorem (Theorem 5) was developed which related the universal coordinate to the averaged coordinate. The limit as $N \to \infty$ was not studied there. Working in the probabilistic setting, a quick study[1] indicates that in this limit, the averaged coordinate forces the universal coordinate to become angle-independent. This is a consequence of Theorem 3 in Section 7.4 of [4]. This may explain why experimentally such fine effects as in the developed theory are not usually reported or observed in the data.

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References

[1] Aman Chawla, Salvatore Morgera, and Arthur Snider. On axon interaction and its role in neurological networks. *IEEE/ACM Transactions on Computational Biology and Bioinformatics*, 2019.

[1] Details are omitted here, being a straightforward application of the strong law of large numbers, under rudimentary assumptions.

[2] Aman Chawla, Salvatore D. Morgera, and Arthur D. Snider. Fields, geometry, and their impact on axon interaction. *Journal of Applied Mathematics and Physics*, 9(4):751–778, 2021.

[3] Aman Chawla. Geometric Relativity of Action Potentials: a Mathematical Approach. March 2024.

[4] Geoffrey Grimmett and David Stirzaker. *Probability and random processes*. Oxford university press, 2020.