# On the Anytime Delay and the Streaming Gaussian Reliability Function

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**Abstract**

In this note the authors reconsider the reliability function of the Gaussian channel in the presence of Gaussian feedback. In this context, they illuminate the relationship between the anytime delay and the overall scheme reliability function.

*This note is dedicated to Prof. Sanjoy Kumar Mitter and Paramahamsa Hariharananda Giri.*

In [1], a four phase scheme was used to communicate over a Gaussian channel in the presence of Gaussian feedback. Suppose the anytime delay $W$ in the final phase has some statistical properties. Using the theory of large deviations [2], we find that if

$$\mathbb{P}(W > \frac{a}{\Delta T} - 1) > 0 \tag{1}$$

where $\Delta T$ is the fixed single-shot transmission duration of the first three phases, and $a$ is a real number, then

$$\Lambda^*(a) > 0 \tag{2}$$

where

$$\Lambda^*(a) = \underset{t \in \mathbb{R}}{sup} \left\{ at - \log \mathbb{E}[e^{t(W+1)\triangle T}] \right\} \tag{3}$$

and $\mathbb{P}((n+1)$-st message acceptance time $> na)$ decays exponentially as $e^{-n\Lambda^*(a)}$. That is, messages are likely to get accepted "soon" if the exponent $\Lambda^*(a)$ is large. In the notation of [1] this would lead to a short expected transmission duration $\bar{\tau}$ and therefore a large streaming reliability function. Thus the synchronizing anytime [3] code's properties are closely tied to the overall streaming performance.

# References

[1] Aman Chawla. Reliability of a gaussian channel in the presence of gaussian feedback. *S.M. Thesis*, 2006.

[2] Geoffrey Grimmett and David Stirzaker. *Probability and random processes.* Oxford university press, 2020.

[3] Anant Sahai. Anytime coding on the infinite bandwidth awgn channel: A sequential semi-orthogonal code. In *Proc. 2005 Conf. Inf. Sci. Syst.* Citeseer, 2005.