On a Unification Problem in Mathematical Neuroscience

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Abstract

In this note, we look at a problem calling for attention in computational neuroscience. Our motivation is to consider the computational and mathematical aspects of the problem, and propose a generalization.

In (Alvarez-Maubecin *et al.*, 2000), the authors extend previous work on 'electrotonic' coupling between neurons in the locus ceruleus to the same type of coupling, but between neurons and glial cells. They find spontaneous oscillations in some glial cell membrane voltages, which are synchronous with the firing of neurons. That the interaction was two-way was confirmed by them when depolarization was induced in the glial potential, and it resulted in changes in the firing rate of the coupled neurons.

In juxtaposition, in current mediated ephaptic coupling (Chawla *et al.*, 2019) it is found that if two axons are coupled, and one of them is stimulated, the coupled axon shows sub-threshold activity. If the coupling is increased in 'strength,' say by modifying the resistivities involved, the coupled axon's activity becomes supra-threshold. That is, it begins to fire normally, and eventually attains synchrony with the 'driving' axon.

Notwithstanding the apparent similarities between axon-axon extracellular current-mediated coupling and neuron-glial gap junction based coupling, there ought to be some important differences between the two. For example, the mechanism in the gap junction case is possibly different from the 'return-path' current based mechanism of axon-axon coupling (see also (Chawla *et al.*, 2021)).

In light of the above, can we propose the construction of a general mathematical model for hetero-cellular coupling, simulate it and study precisely where the two instances of models discussed herein diverge, in a mathematical sense?

References

Alvarez-Maubecin, Veronica, García-Hernández, Fernando, Williams, John T, & Van Bockstaele, Elisabeth J. 2000. Functional coupling between neurons and glia. *Journal of neuroscience*, **20**(11), 4091–4098.

Chawla, Aman, Morgera, Salvatore, & Snider, Arthur. 2019. On axon interaction and its role in neurological networks. *Ieee/acm transactions on computational biology and bioinformatics*.

Chawla, Aman, Morgera, Salvatore D., & Snider, Arthur D. 2021. Fields, geometry, and their impact on axon interaction. *Journal of applied mathematics and physics*, **9**(4), 751–778.