On Metric Perturbations and Channel Capacity: Information Acquisition in Spacetime

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Abstract

In a prior paper [1] we studied the influence of gravitational radiation on the information transmission intricacies of a neurobiological channel. Here we consider the Gaussian channel. We find that there is no impact of metric perturbations on the capacity of said channel. Next we consider the conditions that *would* yield such an impact and how that consequently leads to a relationship between bits gained from the channel and delay incurred while doing so.

1 Separation Between Masses

A simplification often used when dealing with applications of Einstein's field equations of gravitation is linearization. In the linearized theory the equation governing gravitational radiation takes a manageable form. We begin our investigation by noting how linearized gravitational radiation influences the relative positions of two test masses A and B, separated by some distance. This is captured by the following formula from Misner-Thorne-Wheeler [2], Equation (35.15):

$$x_B^{\hat{j}}(\tau) = x_{B(0)}^{\hat{k}}[\delta_{jk} + \frac{1}{2}h_{jk}^{TT}] \tag{1}$$

2 Communication in the Presence of Noise

Consider a communication scheme wherein the channel input is $x(t)$, the channel output is $y(t)$ and the channel noise is $z(t)$. Usually, such a channel is written mathematically as

$$y(t) = x(t) + z(t) \tag{2}$$

However, note that the above equation does not feature any delay required in receiving the transmitted channel input at the destination.

3 Including the Effect of Delay

Motivated by Equation (1), we propose the following modification to $x(t)$:

$$x''(t) = x(t - \gamma). \tag{3}$$

Here $\gamma = \frac{\triangle}{c}$ is the time delay in receiving the signal from the transmitter at the receiver, and $\triangle$ is the nominal separation between the two.

4 Delay Caused by Metric Perturbations

If gravitational radiation is involved, this $\triangle$ will vary with time, based on Equation (1). An example of such a variation is

$$\triangle \to \triangle(1 + \frac{1}{2} A \cos(\omega t)) \tag{4}$$

where A and ω are, respectively, the amplitude and angular frequency of the gravitational radiation (see Equation (35.16) in [2]).

5 Channel Induced by Low Frequency Gravity Waves

If the frequency of the gravitational radiation is very low, then we get the following channel:

$$y''(t) = x(t - \frac{\triangle}{c}(1 + \frac{A}{2})) + N(t) \tag{5}$$

This is a general linear Gaussian channel with C.I.R. (channel impulse response) as a delta function. Specifically,

$$C.I.R.(t) = \delta(t - \frac{\triangle}{c}(1 + \frac{A}{2})) \tag{6}$$

Since the Fourier transform of a delta function is unity, the channel impulse response does not contribute to the channel capacity, and we recover the usual relation for the capacity of the channel, namely, $W \log(1 + SNR)$, derived by Shannon [3]. Hence, the gravitational radiation of the form chosen for illustrative purposes, has no impact on the channel capacity.

6 Time-scaled and -shifted Inputs

Next we will examine *the conditions needed* for gravitational radiation to have an impact on channel capacity. We will work with an example. Suppose we start with the transmitted signal as specified in Equation (3). Our goal will be to express it such that the resulting Gaussian channel has a non-trivial contribution to capacity from the gravitational radiation. We have,

$$x(t - a(\frac{\gamma}{a})) = x(a(s - \frac{\gamma}{a})) \tag{7}$$

We have made a change in the time variable by scaling it:

$$\frac{t}{a} \to s.$$

Since the Fourier transform of the time-scaled version of a function is the scaled version of the frequency-scaled frequency domain signal, therefore having expressed the transmitted signal as in Equation (7), we obtain a factor of a in front of the channel impulse response which is an exponential in the frequency domain. This factor, related to the gravitational radiation's amplitude, means that the time domain channel impulse response will be a scaled and shifted delta function. The height-scaling factor will contribute to the channel capacity formula and we will have obtained a capacity which depends on the gravitational radiation. In summary, when we are able to express the time variable t as $as - \gamma$, then the gravitational radiation amplitude would impact the channel capacity.

7 Relation Between Delay and Information

Thus, we would have the rudiments of a relationship between the time delay γ and the channel capacity in bits per channel use. In other words, various different time delays would lead to different numbers of bits one can gain from the channel per use of the channel. Succinctly, 'bits gained' have been related to 'delay suffered' [4]. We may write this out in a more general way using a functional representation, as

$$\triangle t = f(\triangle I) \tag{8}$$

8 Discussion: Implications for the Nervous System

In conclusion, in this paper we have considered the (non-)impact of metric perturbations on Gaussian channel capacity and have developed a 'time-scale-and-shift' condition under which impact can be obtained. In this latter condition, information acquisition has been related to the time delay incurred while doing so. If we treat the Gaussian channel as a 'representative' channel, we may conclude that flow of information in the nervous system, when 'seen' at various different non-ordinary time scales by entering into 'altered' states of perception, and at non-usual relative times by tuning into others' mindsets, say, is influenced by universal gravitational perturbations in the spacetime metric.

References

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