On Finding the Parity of an Unknown Quantum State

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Abstract

In this note the authors suggest a problem which is a variant on the quantum MAXSAT problem and is inspired by a classical distributed communication and computation problem of Gallager.

We consider the problem of finding the parity (or some simple function) of an unknown quantum state, given N approximate clones of that state. Suppose some algorithm generates a clone of an unknown input state. The clone is "pretty" close to the input state but prevented from being an exact replica by the no-cloning theorem [1]. N such clones are created, one with each "transmitter." Suppose the N transmitters can cooperate fully so that they jointly act as a single N qubit space. Then we perform a quantum random walk step multiple times in conjunction with a reflection operator to explore the nearby state-space, with the goal of stopping when the probability of landing on a state whose parity is the same as that of the unknown state becomes "pretty" high. We are not concerned with how close the final state is to the unknown state; we merely want their parities to match. Next, consider the case when the N "transmitters" cannot cooperate fully. Thus their joint space is not the tensor product space, but a larger noisy space. The noise comes in because they must "communicate" amongst each other to create this joint space. The problem once again is to find a quantum algorithm that finds the parity of the unknown initial quantum state. The algorithm should minimize rounds of communication by being as "distributed" as possible. In this sense this problem is a quantum version of the problem set up classically by Gallager in [2].

References

[1] William K Wootters and Wojciech H Zurek. The no-cloning theorem. *Physics Today*, 62(2):76–77, 2009.

[2] Robert G Gallager. Finding parity in a simple broadcast network. *IEEE Transactions on Information Theory*, 34(2):176–180, 1988.