Knots and Information

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Abstract

In this note, the authors suggest an analogy between a knot operation and a communication channel and therefore propose a link between topology and information theory.

In topology and the study of knots and links, the Jones polynomial is a useful concept [1, 2]. It is an invariant of a knot or link. This means that as the knot is transformed by a specific class of operations, its Jones polynomial remains unchanged. Thus the Jones polynomial helps to "characterize" the knot or link. However, there are certain other more dramatic operations (like "cutting") that don't preserve the Jones polynomial.

Due to its character as an invariant, specifying a Jones polynomial doesn't determine the precise knot configuration, rather a whole class of configurations is indicated. Next, consider that we start with a Jones polynomial, apply an operation and end up with a different Jones polynomial. The operation can then be associated with the "Jones difference polynomial." This has the benefit of making it very clear how a given Jones polynomial will be transformed under an operation.

As a result of these two associations, that of the Jones polynomial with a whole class of configurations and that of an operation with a Jones difference polynomial where the zero-operation Jones difference polynomial corresponds to applying simple isotopies, we can consider an operation to be akin to a communication channel [3, 4] and a Jones polynomial to be akin to a random variable [5]. The analogy can be strengthened if when looking to determine which configuration corresponds to a given Jones polynomial, we pick one up randomly according to an associated probability distribution. We may then be able to study the *capacity* of various Jones difference polynomials, since they would induce an "output" probability distribution on knot configuration space.

Finally, since a Jones difference polynomial is a type of polynomial after all, this is a step towards some sort of capacity notion applied to a generic polynomial and could be mathematically interesting.

References

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