Inverse Perceptrons and Regression

Aman Chawla

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Abstract

In this note the authors present an "inverse" perceptron. The inverse perceptron is seen to have auto-regressive (AR)-like components.

Regression and the Rosenblatt perceptron are usually presented separately in machine learning courses or texts. In this note, we study the inverse perceptron and find that its components can be viewed as auto-regressive processes. Note that

$$w[n+1] = w[n] + \eta(d[n] - y[n])x[n] \tag{1}$$

is the weight update equation for the perceptron. Now treat $d[n]$ as the input and $x[n]$ as the output of the inverse perceptron. After some manipulation and the use of a Taylor series approximation, we find that for small inputs,

$$x[n] = (a/y[n])d[n] + (-a) \tag{2}$$

where

$$a = \frac{w[n] - w[n+1]}{\eta y[n]}. \tag{3}$$

We can view a as a type of AR process. The general AR process can be written [1] as

$$x_t = \sum_{i=1}^{p} a_i x_{t-i} + \epsilon_i \tag{4}$$

where t is an integer, and p is a positive integer. Thus here $w[n]$ corresponds to x_{t-1}, $w[n+1]$ corresponds to x_t, a_1 is $-\frac{1}{\eta y[n]}$ and a_2 is the negative of a_1. One can question if a_1 is "allowed" to be time dependent as in our choice. But a careful examination of Equation (4) indicates that the sum on the right hand side is a discrete convolution between the a process and the past of x. Thus it may be justifiable to allow both the participants of the convolution operation to be temporal processes on an equal footing. We can modify Equation (4):

$$x_t = \sum_{i=1}^{p} b_i x_{t-i} + \epsilon_i \tag{5}$$

where

$$b_i = a_{t-i} \tag{6}$$

In summary, we have represented an inverse perceptron (Equation (2)) as follows:

$$x[n] = \alpha \cdot d[n] + \beta \tag{7}$$

where α and β are AR with time-varying coefficients. In future work we hope to find the capacity of the specific auto-regressive channel given in Equation (7).

References

[1] Ching-Kang Ing. Multistep prediction in autoregressive processes. *Econometric theory*, 19(2):254–279, 2003.