Geometric Application of Bell's Perturbative Theorems

Aman Chawla

May 9, 2024

Abstract

In this paper, the authors enhance the theoretical basis for three dimensional ephaptic synchronization. This is done as an application of the theorems of perturbative coupling, proven by Jonathan Bell, extended to the three-dimensional geometric setting.

Jonathan Bell, in [1], has provided the conditions under which coupled ephaptic propagation takes place on two parallel fibers. In this work we investigate his theorems and extract from them the conditions that would be needed to apply them to the case of three dimensional tracts. These three dimensional tracts were introduced in [2, 3]. The geometry is incorporated by a phenomenologically introduced (and later justified (in [4])) W-matrix and axonal inclinations.

By making a table of correspondence between the variables in [2] and those in Bell's referred work, and changing appropriately to the traveling frame, we get the following observation. The axonal inclinations must be small and, if we assume the inclinations are smaller than say 30 degrees then MATLAB (see Algorithm 1) verifies that the extracellular conductance must be larger than the axoplasmic conductances (for most choices of other parameters), and there must be a linear relation between axoplasmic conductance and capacitance.

These mathematical observations based on Bell's work validate our simulations of the programs developed in 2019 and 2021 wherein, if axonal inclinations were made too large, the output fell out of the boundaries of synchronized or synchronizing propagation.

In conclusion, the use of pre-established theory shows that the work on ephaptic coupling in geometric conditions is also well founded.

References

[1] Jonathan Bell. Modeling parallel, unmyelinated axons: pulse trapping and ephaptic transmission. *SIAM Journal on Applied Mathematics*, 41(1):168–180, 1981.

Algorithm 1 Matlab code for verifying the relations between the conductances and capacitances for coupling to take place.

```
close all;
clear all;
clc;
clf;
G1ax = linspace(0,0.1,10);
theta1 = linspace(0,90,10);
theta2 = linspace(0,90,10);
G2ax = linspace(0,0.1,10);
Gext = linspace(0,0.1,10);
C1 = 3.14e-9;
W11 = 1;
K1 = G1ax.*cosd(theta1)./[Gext+G1ax.*cosd(theta1)+G2ax.*cosd(theta2)]
y = C1./[(G1ax.*(cosd(theta1)).^2).*(1-W11.*K1)]
figure(1)
G1ax = 0.1*rand(1);%linspace(0,0.1,10);
theta1 = 90*rand(1);%linspace(0,90,10);
theta2 = 90*rand(1);%linspace(0,90,10);
G2ax = 0.1*rand(1);%linspace(0,0.1,10);
Gext = linspace(0,0.1,10);
C1 = 3.14e-9;
W11 = 1;
K1 = G1ax.*cosd(theta1)./[Gext+G1ax.*cosd(theta1)+G2ax.*cosd(theta2)]
y = C1./[(G1ax.*(cosd(theta1)).^2).*(1-W11.*K1)]
plot(Gext,y);
figure(2)
G1ax = linspace(0,0.1,10);
theta1 = 90*rand(1);%linspace(0,90,10);
theta2 = 90*rand(1);%linspace(0,90,10);
G2ax = 0.1*rand(1);%linspace(0,0.1,10);
Gext = 0.1*rand(1);%linspace(0,0.1,10);
C1 = 3.14e-9;
W11 = 1;
K1 = G1ax.*cosd(theta1)./[Gext+G1ax.*cosd(theta1)+G2ax.*cosd(theta2)]
y = C1./[(G1ax.*(cosd(theta1)).^2).*(1-W11.*K1)]
plot(G1ax,y)
figure(3)
G1ax = 0.1*rand(1);%linspace(0,0.1,10);
theta1 = 90*rand(1);%linspace(0,90,10);
theta2 = 90*rand(1);%linspace(0,90,10);
G2ax = linspace(0,0.1,10);
Gext = 0.1*rand(1);
C1 = 3.14e-9;
W11 = 1;
K1 = G1ax.*cosd(theta1)./[Gext+G1ax.*cosd(theta1)+G2ax.*cosd(theta2)]
y = C1./[(G1ax.*(cosd(theta1)).^2).*(1-W11.*K1)]
plot(G2ax,y);
```

Algorithm 2 Continuation of Algorithm 1.

```
figure(4)
G1ax = 0.1*rand(1);%linspace(0,0.1,10);
theta1 = linspace(0,90,10);
theta2 = 90*rand(1);%linspace(0,90,10);
G2ax = 0.1*rand(1);%linspace(0,0.1,10);
Gext = 0.1*rand(1);%linspace(0,0.1,10);
C1 = 3.14e-9;
W11 = 1;
K1 = G1ax.*cosd(theta1)./[Gext+G1ax.*cosd(theta1)+G2ax.*cosd(theta2)]
y = C1./[(G1ax.*(cosd(theta1)).^2).*(1-W11.*K1)]
plot(theta1,y);
figure(5)
G1ax = 0.1*rand(1);%linspace(0,0.1,10);
theta1 = 90*rand(1);%linspace(0,90,10);
theta2 = linspace(0,90,10);
G2ax = 0.1*rand(1);%linspace(0,0.1,10);
Gext = 0.1*rand(1);%linspace(0,0.1,10);
C1 = 3.14e-9;
W11 = 1;
K1 = G1ax.*cosd(theta1)./[Gext+G1ax.*cosd(theta1)+G2ax.*cosd(theta2)]
y = C1./[(G1ax.*(cosd(theta1)).^2).*(1-W11.*K1)]
plot(theta2,y);
%%%% figure(6)
ratio = 0.5;
rf = 1.27e8;
r0 = ratio*rf;
gm = 5.6e-9; % per Ohm-cm (both)
cnd = 3.14e-9; % F/cm
gnd = 0;
theta1 = 5;
theta2 = 6;
G2ax = linspace(0,0.9e-8,10);
G1ax = G2ax;
Gext = 2*G1ax;
c2 = G2ax - (G2ax).^2./[Gext+G1ax+G2ax]
plot(G2ax,c2)
```

[2] Aman Chawla, Salvatore Morgera, and Arthur Snider. On axon interaction and its role in neurological networks. *IEEE/ACM Transactions on Computational Biology and Bioinformatics*, 2019.

[3] Aman Chawla, Salvatore D. Morgera, and Arthur D. Snider. Fields, geometry, and their impact on axon interaction. *Journal of Applied Mathematics and Physics*, 9(4):751–778, 2021.

[4] Aman Chawla and Salvatore Domenic Morgera. On the geometry of interacting axon tracts in relation to their functional properties. *bioRxiv*, 2022.