FSO Channel Design with Halo-Inclusive Source

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Abstract

मुक्त-अंतरिक्ष ऑप्टिकल (FSO) संचार में प्रायः या तो एक बिंदु-स्रोत आधारित विवर्तन (Airy) मॉडल या केवल Gaussian बीम मॉडल का उपयोग किया जाता है। वास्तविक प्रेषक (transmitters) न तो पूर्ण बिंदु-स्रोत होते हैं और न ही आदर्श Gaussian—उनमें एक विस्तृत स्रोत-आधारित Gaussian हलो तथा छिद्र-अवरोधन (aperture clipping) से उत्पन्न Airy रिंगों का संयुक्त प्रभाव उपस्थित रहता है। इस कार्य में एक **एकीकृत halo-inclusive** ढाँचा प्रस्तुत किया गया है, जिसमें दोनों योगदानों को एक ही स्केलर क्षेत्र मॉडल में सम्मिलित कर पूर्ण ऊर्जा संरक्षण सुनिश्चित किया गया है।

प्रस्तुत मॉडल में Gaussian कोणीय फैलाव एवं Airy विवर्तन गुणक का संयुक्त उपयोग कर कुल शक्ति, धुरी-पर तीव्रता तथा चैनल गेन के लिए बंद-आकार सूत्र निकाले गए हैं। दो सीमित स्थितियों — (1) विस्तृत-स्रोत Gaussian-प्रधान तथा (2) विवर्तन-प्रधान बिंदु-स्रोत — का विश्लेषण कर यह दर्शाया गया है कि पारंपरिक inverse-square पथ-हानि नियम केवल विशिष्ट ज्यामितीय स्थितियों में ही लागू होता है और व्यावहारिक प्रणालियों में महत्वपूर्ण विचलन उत्पन्न हो सकता है।

एकीकृत मॉडल **अधिक यथार्थवादी FSO लिंक डिज़ाइन, अनुकूलन, तथा प्रदर्शन-अनुमान** में सहायक है और भविष्य के कार्य हेतु नए मार्ग खोलता है, जिनमें वायुमंडलीय अशांति, पॉइंटिंग त्रुटियाँ, MIMO FSO और विस्तृत-हलो कैप्चर शामिल हैं।

Abstract

Free-space optical (FSO) communication models typically assume either a diffraction-limited point source truncated by a circular aperture or an ideal Gaussian beam representing an extended emitter. In practice, real transmitters exhibit both finite source extent and aperture-induced diffraction, producing a characteristic halo that significantly alters far-field intensity distribution and received power. This paper presents a unified halo-inclusive framework that rigorously incorporates both effects within a single scalar field model by multiplying a Gaussian angular envelope with the classical Airy pattern and enforcing energy conservation. Closed-form expressions for total power, on-axis intensity, and channel gain are derived, together with analytical results in two limiting regimes: Gaussian-dominated extended sources and diffraction-dominated point-like sources. The general case is formulated for numerical evaluation. The analysis demonstrates that the conventional inverse-square path-loss law emerges only under restricted geometric conditions and may deviate substantially otherwise. The unified model enables more accurate FSO link design, optimization, and performance prediction.

1 Introduction

> Science is a universal language, speaking to all who are willing to listen, regardless of race, religion, or nationality. – C. V. Raman

It is common to observe LEDs with a "spherical halo" around them, without stopping to wonder about their modeling. Free-space optical (FSO) communication theory frequently assumes an ideal point source truncated by a circular aperture, yielding the familiar Airy diffraction pattern, or a pure Gaussian beam originating from a diffraction-limited laser embedded in a Gaussian aperture [4]. In practice, neither extreme fully describes real transmitters: semiconductor lasers possess finite source size and gain-guided lateral modes, while high-power LEDs have extended emitting areas many times larger than the wavelength. Crucially, no physical source is ever observed without some form of halo—whether a broad Gaussian halo from source extent or narrow diffraction rings from aperture clipping [5]. Despite this, the overwhelming majority of FSO channel models in the literature treat these two contributions separately, or simply ignore the halo altogether.

A systematic treatment that retains both effects simultaneously—termed here a "halo-inclusive" model—remains surprisingly sparse. A few works incorporate extended-source Gaussian spreading, and many more treat diffraction-limited Airy patterns, but combined models that conserve total radiated power while preserving the correct functional form of each contribution are rare. This gap matters: the interplay between source size and aperture truncation governs the far-field beam divergence, the fraction of power contained in the central lobe versus rings/halo, and ultimately the received power and link margin in practical systems.

The present work closes this gap by starting from the exact scalar field of a uniformly illuminated circular aperture with an additional Gaussian angular envelope, rigorously normalising the radiated power, and deriving closed-form channel gains in the two practical limiting cases. We further provide the general expression for arbitrary source-aperture combinations and highlight why the inverse-square path loss law emerges only under specific (and often misstated) conditions.

The remainder of this paper is organized as follows. Section 2 presents the halo-inclusive scalar field model. Section 3 establishes energy conservation and derives the intensity normalization constant. Section 4 and Section 5 develop the two limiting cases—Gaussian-dominated and diffraction-dominated propagation—and their channel gain expressions. Section 6 generalizes the result to arbitrary parameter regimes. Section 7 discusses implications for practical FSO link design and directions for further work.

Table of Notation

The adjacent Table 1 lists the principal symbols and their meanings as used in the present work.

Table 1: Table of Notation

Symbol	**Description**
L	Propagation distance (link range)
λ	Wavelength of light
$k = 2\pi/\lambda$	Wave number
a	Radius of the transmitter circular aperture
a_{RX}	Radius of the receiver circular aperture
w_0	Effective source radius (at transmitter plane)
θ	Off-axis angle from the beam centerline
$\theta_G = \lambda/(2\pi w_0)$	Gaussian $1/e$ intensity half-angle (far-field divergence half-angle of the extended source)
θ_G^2	Appears in exponents; note the specific factor of 2 used in the paper
P_T	Total transmitted optical power
P_R	Total received optical power
$E(r, \theta, t)$	Far-field scalar electric field
E_0	Electric field amplitude constant (before normalisation)
$J_0(x)$, $J_1(x)$	Bessel functions of the first kind (order 0 and 1)
$I(L, \theta)$	Far-field intensity distribution (W/m²)
$I_0 = \frac{c\epsilon_0}{2}E_0^2$	Unnormalised on-axis intensity constant
c	Speed of light in vacuum
ϵ_0	Vacuum permittivity
ω	Angular frequency of the optical field
$r_\perp$	Radial coordinate in the receiver plane
$\mathcal{N}$	Normalisation integral ensuring total radiated power equals P_T
$H(L)$	General halo-inclusive channel gain (P_R/P_T)
$H_G(L)$	Channel gain in the Gaussian-dominated (extended-source) limit
$H_D(L)$	Channel gain in the diffraction-dominated (point-like source) limit
ξ	Dummy integration variable (often $\xi = ka\theta$ or $\xi = kar_\perp/L$)

2 Electric Field of a Halo-Inclusive Source

We begin with the far-field electric field of a circular transmitter aperture of radius a whose exit-plane illumination is limited both by hard-edged clipping and by the finite angular spread of the source itself. The resulting field in the scalar approximation is the product of a Gaussian envelope and the classic Airy pattern [1]:

$$E(r,\theta,t) = \frac{E_0}{r}\exp\left(-\frac{\theta^2}{\theta_G^2}\right)\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right]e^{i(kr-\omega t)}, \tag{1}$$

where $\theta_G = \lambda/(2\pi w_0)$ is the $1/e$ intensity half-angle of the Gaussian component determined by the effective source radius w_0, $k = 2\pi/\lambda$, and the remaining symbols have their usual meaning [2].

The time-averaged intensity then follows immediately:

$$I(L,\theta) = \frac{c\epsilon_0}{2}|E(L,\theta)|^2 = I_0\frac{1}{L^2}\exp\left(-\frac{2\theta^2}{\theta_G^2}\right)\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right]^2, \tag{2}$$

with $I_0 = \frac{c\epsilon_0}{2}E_0^2$.

Having established the intensity distribution, we now enforce energy conservation to relate I_0 to the total transmitted power P_T.

3 Energy Conservation and Intensity Normalisation

The total transmitted power is recovered by integrating the intensity over the forward hemisphere:

$$P_T = \int_0^{2\pi}\int_0^{\pi/2} I(L,\theta)L^2\sin\theta\,d\theta\,d\phi. \tag{3}$$

Under the paraxial approximation ($\sin\theta \approx \theta$, valid for practically relevant FSO links), this reduces to

$$P_T = 2\pi I_0\int_0^{\infty}\exp\left(-\frac{2\theta^2}{\theta_G^2}\right)\left[\frac{2J_1(ka\theta)}{ka\theta}\right]^2\theta\,d\theta. \tag{4}$$

Defining the normalisation integral

$$\mathcal{N} \equiv \int_0^{\infty}\exp\left(-\frac{2\theta^2}{\theta_G^2}\right)\left[\frac{2J_1(ka\theta)}{ka\theta}\right]^2\theta\,d\theta, \tag{5}$$

the correctly normalised intensity becomes

$$I(L,\theta) = \frac{P_T}{2\pi\mathcal{N}L^2}\exp\left(-\frac{2\theta^2}{\theta_G^2}\right)\left[\frac{2J_1(ka\theta)}{ka\theta}\right]^2. \tag{6}$$

The value of $\mathcal{N}$ depends on the ratio $ka\theta_G$; analytical solution is possible only in limiting cases, to which we now turn.

4 Limiting Case 1: Gaussian-Dominated (Extended Source)

When the source is large compared to the wavelength ($w_0 \gg \lambda$), the diffraction pattern collapses to a near-δ-function, and $\mathcal{N} \to 1/(2\theta_G^2)$. The intensity simplifies to the familiar extended-source form:

$$I(L,\theta) = \frac{P_T}{\pi(L\theta_G)^2}\exp\left(-\frac{\theta^2}{\theta_G^2}\right). \tag{7}$$

Received power collected by a circular aperture of radius a_{RX} at distance L is

$$P_R = P_T\left[1-\exp\left(-\frac{2a_{RX}^2}{L^2\theta_G^2}\right)\right]. \tag{8}$$

Thus the channel gain in the Gaussian-dominated regime is

$$\boxed{H_G(L) = 1 - \exp\left(-\frac{2a_{RX}^2}{L^2\theta_G^2}\right)}. \tag{9}$$

5 Limiting Case 2: Diffraction-Dominated (Effectively Point-Like Source)

When the Gaussian spread is negligible ($\theta_G \to 0$ or equivalently a very large aperture relative to source size), the pattern becomes a pure Airy disc plus rings. Substituting the known integral $\int_0^\infty [2J_1(x)/x]^2 x\,dx = 1$ yields the correctly normalised on-axis intensity

$$I(L,\theta) = \frac{P_T k^2 a^2}{\pi L^2}\left[\frac{2J_1(ka\theta)}{ka\theta}\right]^2. \tag{10}$$

The power incident on the receiver aperture is then

$$P_R = P_T \int_0^{kaa_{RX}/L} \left[\frac{2J_1(\xi)}{\xi}\right]^2 \xi\,d\xi = P_T\left[1 - J_0^2\left(\frac{kaa_{RX}}{L}\right) - J_1^2\left(\frac{kaa_{RX}}{L}\right)\right], \tag{11}$$

where we have used the standard encircled-energy identity for the Airy pattern. The diffraction-dominated channel gain is therefore

$$\boxed{H_D(L) = 1 - J_0^2\left(\frac{2\pi a a_{RX}}{\lambda L}\right) - J_1^2\left(\frac{2\pi a a_{RX}}{\lambda L}\right)}. \tag{12}$$

Both limiting cases recover the expected physical behaviour: $H \to 1$ for arbitrarily large receiver apertures and the classic $1/L^2$ far-field scaling when a_{RX} is small compared to the beam footprint.

6 General Halo-Inclusive Channel Gain

For arbitrary ratios of source size and aperture truncation, the full channel gain retains the form

$$\boxed{H(L) = \frac{1}{\mathcal{N}} \int_0^{a_{RX}} \exp\left(-\frac{2r_\perp^2}{L^2\theta_G^2}\right)\left[\frac{2J_1(kar_\perp/L)}{kar_\perp/L}\right]^2 2\pi r_\perp\,dr_\perp}, \tag{13}$$

with $\mathcal{N}$ defined in Eq. (5). This expression must generally be evaluated numerically but poses no conceptual difficulty.

7 Discussion and Path Forward

A key insight emerging from both limiting cases is that the inverse-square path loss law $P_R \propto 1/L^2$ holds only when the receiver aperture subtends a small solid angle relative to the beam—exactly the regime where most simplified FSO models are applied. In all other cases, the channel gain deviates significantly from pure geometric loss.

Far from being the final word, the halo-inclusive framework presented here serves as a rigorous starting point for a vast array of extensions that were previously ambiguous or inconsistent:

- Atmospheric turbulence effects on beams with realistic halo structure [3],
- Pointing error statistics when the halo power distribution is non-Gaussian [6],
- MIMO FSO with aperture arrays and partial halo capture,

- Non-line-of-sight UV scattering channels dominated by broad halos,
- Optimisation of transmitter aperture radius a given fixed source divergence θ_G.

Virtually every existing FSO investigation can be revisited and strengthened by replacing idealised point/Gaussian assumptions with the combined model derived above. The halo is not a nuisance to be approximated away—it is an inherent feature of all real optical sources, and treating it explicitly opens a rich, physically accurate landscape for future theoretical and experimental work.

Acknowledgments

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