Deformations in axons, and their threshold.

Aman Chawla

December 17, 2023

Abstract

In this note, the authors provide an explanation for a potential relation between the excitability of neuronal axons and their tortuosity. This is done via fundamental considerations of calculus on manifolds.

1 Introduction

The Hodgkin Huxley equation sets the stage for a mathematical approach to understanding nerve axons. They provide a derivation along with empirical or phenomenological aspects in their classic papers from 1952 [1, 2]. In [3] the authors look at tortuous and coupled Hodgkin-Huxley nerves. The paper incorporates a three dimensional geometrical arrangement into the standard formalism.

2 The Link

Geometry and physics have long had numerous interlinkages [4]. Even the theory of general relativity of Albert Einstein, is at heart geometric in nature, so much so that it is also called geometrodynamics [5]. One can speculate unendingly about the reasons for this close connection. However, in this paper we will not pursue this direction. Instead we provide a simple link between geometry and biophysics that seems to suggest a mechanism for the increased sensitivity (lower firing threshold) of tortuous axons [6] as opposed to linear ones.

3 An Application

Consider a vector space A (like R^2), with a norm N. Suppose we modify the norm such that it applies different factors to the two basis vectors of A. This would result in a skewed version of A. This can be useful to model excitable membranes when they are not placed on a cylinder, but on a tortuous cylinder. The space and time coordinates of the membrane are then deformed in relation to the original coordinates, as above. This results in a skewed action potential. In certain cases of specific skew, the action potential shape may be deformed such that threshold is reached faster. This seems to imply that the tortuous version of the axon is more excitable due to fundamental geometric considerations, rather than electrochemical reasons. More investigations relating geometry to nerve excitability are required to elucidate the connection exposed herein.

References

[1] Alan L Hodgkin and Andrew F Huxley. A quantitative description of membrane current and its application to conduction and excitation in nerve. *The Journal of physiology*, 117(4):500, 1952.

[2] Alan Lloyd Hodgkin and Andrew Fielding Huxley. Propagation of electrical signals along giant nerve fibres. *Proceedings of the Royal Society of London. Series B-Biological Sciences*, 140(899):177–183, 1952.

[3] Aman Chawla and Salvatore Domenic Morgera. On the geometry of interacting axon tracts in relation to their functional properties. *bioRxiv*, 2022.

[4] Michael Atiyah, Robbert Dijkgraaf, and Nigel Hitchin. Geometry and physics.

[5] Kip S Thorne, Charles W Misner, and John Archibald Wheeler. *Gravitation*. Freeman, 2000.

[6] Stamatios N Sotiropoulos and Peter N Steinmetz. Assessing the direct effects of deep brain stimulation using embedded axon models. *Journal of neural engineering*, 4(2):107, 2007.