Behavioral Gravity

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Agenda

- Review
- Theorem Preliminaries
- Theorem
- Motivation
- Next steps
- References

Background: Behavioral Control Theory - 1

- Jan Willems invented behavioral control theory
- In the following we will take a sneak peek at his method of taking a nonlinear dynamical system and linearizing it about the equilibrium point
- Consider a (non)linear dynamical system, such as the axon:

$$C\frac{\partial V(x,t)}{\partial t} = k\frac{\partial^2 V(x,t)}{\partial x^2} - gV(x,t) + I$$

- In the following, ignore that there is an injected current I
- Taking all the terms to the LHS, we obtain (next slide):

Background: Behavioral Control Theory - 2

- $C\frac{\partial V(x,t)}{\partial t} - k\frac{\partial^2 V(x,t)}{\partial x^2} + gV(x,t) = 0\ldots\{1\}$
- We can think of this as a function f of 3 variables: $V(x,t), \frac{\partial V(x,t)}{\partial t}, \frac{\partial^2 V(x,t)}{\partial x^2}$.
- We are mindful of the fact that each variable is in turn a function of x and t
- An equilibrium point would be a point $V(x,t) = a^*$ such that when f is evaluated at $(a^*, 0,0)$ we get 0.
- So at equilibrium, in addition to f, $V(x,t)$ is also not changing.

Background: Behavioral Control Theory - 3

- Next, we will linearize the equation $f = 0$ about the equilibrium point. **Note**: this equation was specialized in {1}.
- Since it is an equilibrium point, the first term in the Taylor series expansion is 0
- The higher terms are non-linear
- So only the second term is to be investigated
- There are two more papers that [1] refers to for this linearization
- So we will not investigate that more
- We will accept the given linearization since it makes sense (mostly)

Background: Behavioral Control Theory - 4

- We will accept the given linearization since it makes sense (mostly)
- Some of the important related issues that come up include differentiation in multivariable calculus, limits of a function of more than one variable etc.
- He obtains,

$$R_0\Delta + R_1\frac{d\Delta}{dt} + R_2\frac{d^2\Delta}{dt^2} + \cdots + R_L\frac{d^L\Delta}{dt^L} \ldots\ldots (*)$$

- The Rs are nothing but partial derivatives with respect to all the $L = 3$ arguments of f
- Such linearizations are familiar from Taylor series expansions of which only the lower than second order terms are kept.
- Then, assuming that $\mathrm{f}(a^*, 0,0) = 0$ is a stabilizable system, he writes down a control law which which stabilizes (*)

Theorem [4]

- We want to discuss the following theorem in order to delve deeper into the linearization discussed last time.
- The theorem itself states that: **Consider the system {1} and assume that at the argument (0,0,0), the system evaluates to 0. Then {*} defines the linearization of {1} if the constant rank conditions [2, Proposition 2.6] are satisfied around 0.**
- [4] contains the proof of this theorem.

Theorem [4] Preliminaries

- Here we will discuss the `constant rank condition' from Proposition 2.6 of [2].
- But first, we quote van der Schaft [2], ``The main result of this paper is that, under constant rank assumptions, we can always **locally transform a general system (1.3) into an equivalent system** not involving any latent variables. Furthermore, we will show how locally this equivalent system without latent variables can be transformed into a reduced form which is more amenable for analysis, in particular for realization purposes.''

Motivation - 1

Straumann [3] states, ``It is a mathematical fact that in a Lorentz manifold one can introduce (normal) coordinates such that, at a given point p, the metric takes the normal form $g_{\mu\upsilon}(p) = diag(-1,1,1,1)$, and in addition its components have vanishing first derivatives: $g_{\mu\upsilon\lambda}(p) = 0$. Relative to such a coordinate system, the gravitational field is **locally transformed away** (up to higher order inhomogeneities). The kinematical structure of GR thus permits locally inertial systems. According to the equivalence principle, the laws of SR (for example, Maxwell's equations) should hold in such systems. This requirement permits one to formulate the non-gravitational laws in the presence of a gravitational field. Mathematically this amounts to the substitution of ordinary derivatives by covariant derivatives. The equivalence principle thus prescribes – up to certain ambiguities – the coupling of mechanical, electromagnetic and other systems to external gravitational fields. This corner stone enabled Einstein to derive important results, such as the gravitational redshift of light, long before the theory was completed.''

Motivation - 2

- Compare the quotations by van der Schaft and Straumann (only the bold portions)
- Question: Is the gravitational field somehow a `latent variable' in a general system?
- Task: Connect the two quotations to learn about gravitational systems and potentially connect them to quantum system via Willems behavioral control and then the James-Gough paper.
- Essentially, we are wondering if the field equations of gravity or a simpler version can be treated as a behavioral system in the context of control theory.

Next Steps

- We didn’t go into the constant rank condition but did provide motivation for studying it in detail
- To study general relativity and quantum physics simultaneously is always our aim
- If we can think of quantum logic devices as being controlled in the presence of gravitational radiation, it will connect with [5]

References

1. Willems, J. C. (1997). *On interconnections, control, and feedback*. IEEE Transactions on Automatic Control, 42(3), 326-339. Section 12.
2. Van der Schaft, A. J. *Transformations of nonlinear systems under external equivalence*. In New Trends in Nonlinear Control Theory: Proceedings of an International Conference on Nonlinear Systems, Nantes, France, June 13–17, 1988, pp. 33-43. Berlin, Heidelberg: Springer Berlin Heidelberg, 2006.
3. Straumann, Norbert. *General relativity*. Springer Science & Business Media, 2012.
4. Willems, J. C., and A. J. Van Der Schaft. *Remarks on the linearization around an equilibrium*. In Nonlinear Control Systems Design 1992, pp. 141-142. Pergamon, 1993.
5. Chawla, Aman, and Salvatore Domenic Morgera. *Mechanism of Universal Quantum Computation in the Brain*. arXiv preprint arXiv:2211.07353 (2022).