An Application of Mitter-Young Theory to Hurwitz Detection

A. Chawla

September 17, 2025

Abstract

In this proposal, the author suggests an application for the theory of integration proposed by Mitter and Young.

In his MIT thesis with Peter Shor [1], Kim discusses codes over Hurwitz surfaces and applies the theory to quantum codes. Suppose a qubit string was encoded using a Hurwitz code in Kim's fashion, then sent over a depolarizing quantum completely positive trace preserving map [2]. The resulting qubit string, modified due to the depolarization, must be subject to some form of decoding. In classical decoding theory, matched filtering is used where the received codeword or symbol string is projected onto the known codeword strings. The received string is decoded to the closest match.

In the quantum setting one way would be to project the received state onto the known codeword subspaces using a suitable operator and determine where the projection was maximum and decode to that codeword. In a generalized version, the projection can be seen as a type of integration and then we can perform integration with respect to a more general measure or operators. This is where Mitter-Young theory [3] comes in and helps us.

References

[1] Isaac H. Kim, Quantum Codes on Hurwitz Surfaces, Ph.D. Thesis, Massachusetts Institute of Technology, 2007, Supervised by Peter W. Shor, Available at: `https://dspace.mit.edu/handle/1721.1/40917`.

[2] Michael A. Nielsen and Isaac L. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, 2000, ISBN: 9780521635035.

[3] Sanjoy K. Mitter and Stephen K. Young, "Integration with respect to operator-valued measures with applications to quantum estimation theory," *Annali di Matematica Pura ed Applicata*, vol. 137, no. 1, pp. 1–39, 1984. **https://doi.org/10.1007/BF01789387**.