

A Remark on Ampliation in Probability and Philosophy

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Consider the definition (Definition 3.1 in [1]) of an adapted process in the quantum stochastic calculus:

Let $S \subset \mathbf{h}$ be an admissible subspace and D a linear manifold in R . Let $F = (F(t) : t \geq 0)$ be a family of operators in h such that for arbitrary $t > 0$, $F(t)$ is the ampliation¹ to $D \otimes G_t \otimes h_t$ of an operator in h_t with domain $D \otimes G_t$. Then F is called an adapted process based on (D, S) .

Also consider side-by-side the definition of a filtration in classical stochastic process theory:

Let $(\Omega, \mathbf{Z})$ be a space with a σ -field. A family $\mathbf{Z}_t : t \geq 0$ of sub- σ -fields of $\mathbf{Z}$ is called a filtration if $\mathbf{Z}_s \subset \mathbf{Z}_t$ if $s \leq t$ [2].

Next consider the definition of adaptation in the classical setting:

A stochastic process X is adapted to the filtration $\mathbf{Z}_t$ if, for each $t \geq 0$, X_t is an $\mathbf{Z}_t$ -measurable random variable [2].

A comparison of the above quoted statements shows the remarkable similarity in how Hudson and Parthasarathy defined the quantum filtration and the classical definition, presumably much older. This also gives us a hint that if we are to do research in the quantum setting, then we can carefully eke out the meaning of a quantum statement by comparison with the classical equivalent. For starters, we would like to see if a quantum version of [3] can be created.

As a concluding remark, we would like to make a brief note that de-dicto and de-re distinctions in philosophical modal logic are similar to the situation when a single statement (such as “some man must be mortal”) can have two distinct meanings – a composite one and a divisive one [4]. This comparison doesn’t seem useful right away, but there is something haunting about it, since it leads us to question how, even in that stratosphere of precision called modal logic, such ambiguities can exist. These considerations of [4] are related to the term ampliation used in the first quotation above and need to be deeply understood, since one meaning can be viewed as an extension of the other one².

¹extension or addition to what is previously defined.

²only a composite thing can be divided.

References

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- [4] G. Priest and S. Read, “Ockham’s rejection of ampliation,” *Mind*, vol. 90, no. 358, pp. 274–279, 1981.