A Personal Finance PDE Whose Aggregation Yields the Black–Scholes Equation

A. Chawla*

July 8, 2025

Abstract

In this note, we propose a micro-level stochastic partial differential equation (PDE) to model the evolution of individual personal wealth over time. The model captures multiplicative growth and uncertainty in assets such as savings, equities, and deposits. We demonstrate that, under a mean-field aggregation and central limit scaling over a large population, the collective dynamics converge to the classical Black–Scholes PDE for option pricing. This formulation offers a bottom-up perspective on financial market models, reinterpreting Black–Scholes as the emergent limit of household-level financial behavior.

1 Model Setup

Let $w_i(t)$ denote the wealth of individual i at time t. We model wealth evolution using geometric Brownian motion:

$$dw_i(t) = \mu w_i(t)\,dt + \sigma w_i(t)\,dW_i(t), \quad i = 1, \ldots, N$$

where μ is the drift (e.g., expected return), σ is the volatility (market uncertainty), and $W_i(t)$ are independent Brownian motions. Define the empirical density $u(w,t)$ of wealth in the population. The associated Fokker–Planck equation is:

$$\frac{\partial u}{\partial t} = -\frac{\partial}{\partial w}[\mu w u] + \frac{1}{2}\frac{\partial^2}{\partial w^2}[\sigma^2 w^2 u]$$

This governs the probability density evolution of wealth among individuals.

*An LLM was used for drafting and brainstorming.

2 Aggregation and Central Limit Convergence

We consider a contingent claim $\phi(w(T))$ on an individual's wealth at terminal time T. Let $V(w,t) = \mathbb{E}[\phi(w(T)) \mid w(t) = w]$. Then, under standard assumptions (independence, identical dynamics, sufficient smoothness of ϕ), $V(w,t)$ satisfies the Kolmogorov backward equation:

$$\frac{\partial V}{\partial t} + \mu w \frac{\partial V}{\partial w} + \frac{1}{2}\sigma^2 w^2 \frac{\partial^2 V}{\partial w^2} = 0$$

This is identical in form to the Black–Scholes PDE, but derived here as a limit of aggregated (over the population) personal financial dynamics.

3 Interpretation and Outlook

Our formulation shows that the Black–Scholes PDE can be viewed as a central limit result arising from stochastic personal finance trajectories. This paradigm shift opens the door to modeling financial markets from agent-level principles, aligning macroeconomic finance with personal wealth evolution and behavioral heterogeneity. Further extensions may incorporate consumption, taxation, risk aversion, or inter-agent interaction (mean-field games), offering a richer bridge between microfinance and classical financial theory.