A Note on the Mass-Position Uncertainty Principle

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Abstract

In this note the authors study the consequence of a non-commutativity between the mass and position operators. They find that the corresponding uncertainty relation lower bound can go from 0 to as large as possible, depending upon the mean position of the particle.

Let $\hat{x}$ be the position operator of a quantum particle and let $\hat{m}$ be the mass operator of the particle. Suppose the commutation relation is

$$[\hat{x}, \hat{m}] = i\hbar\hat{g}(\hat{x}). \tag{1}$$

Assuming that $\hat{g}(\hat{x})$ takes the form

$$\hat{g}(\hat{x}) = \hat{x}, \tag{2}$$

the corresponding uncertainty relation takes the form

$$(\triangle\hat{x})^2(\triangle\hat{m})^2 \geq \frac{1}{4}\hbar^2|\langle\hat{x}\rangle|^2. \tag{3}$$

In particular, the mean position of the particle appears on the RHS bound in a squared form. Thus, the RHS lower bound can go from 0 to infinity and as it proceeds in this vein, the difficulty in a simultaneous measurement of the position and the mass of the particle increases. When the mean position is 0, the mass and position can be measured precisely and simultaneously. However, when the mean position increases, this no longer is feasible. So for example if we try to measure the position accurately to place the apparatus, the apparatus becomes less accurate.

This would have consequences for general relativity where mass influences the curvature of spacetime. So, as a mass goes farther from the origin, and we know its position fairly well to some accuracy, the mass itself becomes fuzzy and spacetime's curvature becomes fuzzy consequently, thereby introducing uncertainty in the trajectories of other masses in the same ambience. This could be tested in a 1919 like experiment and we would expect a quantum correction over and above Einstein's correction.