A Mathematical Model for the Vector Ephaptic Perceptron

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Abstract

Rosenblatt's perceptron was a breakthrough in the mathematical understanding of learning and related higher cognitive functions. In the present paper, the authors extend Rosenblatt's model to the vector setting which enables the modeling of coupling between perceptrons. The form of the coupling suggests a geometric formulation as well which connects the present work to prior work on metric perturbations in axon tracts.

1 Introduction

There are several questions that one would like to answer with regards to views of the brain. First of all, in (Chawla *et al.*, 2019), the authors classified the analysis of the brain into two layers - the data layer and the physical layer. At the physical layer level they proposed in that paper that there is a geometric impact on ephaptic interaction between neurons. In (Chawla *et al.*, 2021) the authors continued their physical layer analysis by extending it to electric fields, and additionally proposed a computation of the information-carrying capacity of the physical layer, thereby initiating the data layer analysis.

In this paper, the authors further the investigation of the brain on the basis of the presence of these two layers. Specifically, they modify the physical layer model of a neural Rosenblatt's perceptron (Rosenblatt, 1958) collection by introducing ephaptic coupling between the members of the set. The authors' investigation raises several questions:

1. The purpose of differential ephaptic coupling seems to be to enable synapse-like features even in the axons. This raises the question if there are other structures in the brain which can be so considered.

2. Synapses allow memory storage. A related question is therefore that if such synapse-like structures are widespread, then memory may be more distributed throughout the organism's nervous system than previously thought, and the next question is one of verification or validation.

3. In terms of spiking activity, coupling leads to synchronization between neural axons within a coupled-tract. A completely unaddressed question therefore is: what is the role of synchronization in the machine learning models that are used to-day?

The above questions are not explicitly addressed in this paper, but they can guide our thinking. The paper is organized as follows. In Section 2, we review the mechanism of ephaptic synchronization. In Section 3 we specify the modeling innovation and in Section 4 we conclude with a discussion.

Notation

Table 1 details the notation used in this paper.

Table 1: Symbols and Their Meanings

Serial Number	Symbol	Meaning
1	$v_{\{1,2,i\}}$	post-summation voltage of perceptrons $1, 2, i$
2	$\tilde{v}_{\{1,2,i\}}$	post-summation ephaptic voltage of perceptrons $1, 2, i$
3	$\lambda_{\{1,2\}}, \lambda_{ki}$	coupling parameters
4	$w_{\{1i,2i\}}$	synaptic weights on perceptrons $1, 2$
5	$x_{\{1i,2i\}}$	input data sent to perceptrons $1, 2$; components of the corresponding vectors
6	$b_{\{1,2\}}$	bias term in perceptrons $1, 2$

2 Review of Ephaptic Synchronization

Two visual exhibits, A and B, will clarify the synchronization dynamics in coupled axons. Exhibit A is Figure 1, which shows three axons coupled via currents. The axons' action potentials are gradually beginning to synchronize in their trains. For example, the initial temporal gap between spikes (considering the rising phase) on axon 2 and axon 1 is 0.00116 seconds whereas the final gap is shorter, at 0.000764 seconds.

Exhibit B is Figure 2, from (Chawla *et al.*, 2019), which shows, in much detail, how perfect phase synchronization gradually sets in, in two coupled axons of differing diameters.

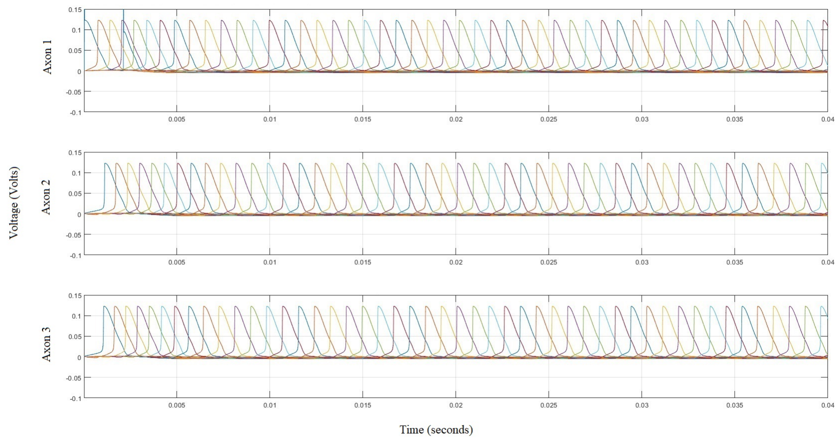

Figure 1: Exhibit A: Three axons in an ephaptically current-coupled tract undergoing gradual phase synchronization.

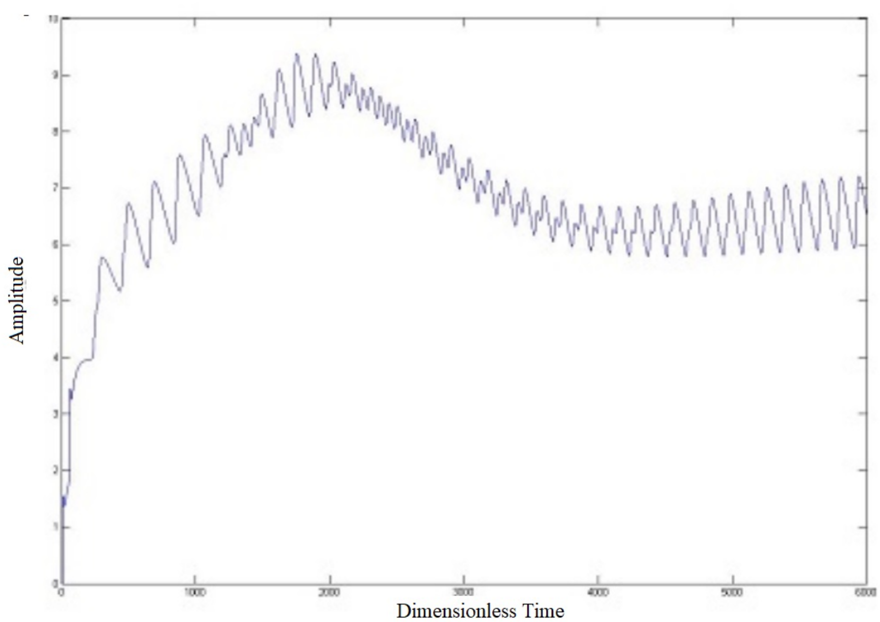

Figure 2: Exhibit B: Perfect phase synchronization between two axons whose diameters differ by 10 percent. The figure is sourced from (Chawla *et al.*, 2019)
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3 The Proposed Model

The regular perceptron consists of incoming data which is nodally-multiplied by so-called synaptic weights (Haykin, 2010). This weighted data is then collected centrally via a summation operation. The summation output is then hard-limited to generate the final output of the neuron. There is also a bias term which is often included as one of the nodes on the input side. The notation used in the following analysis is succinctly explained in Table 1.

Consider the post-summation variables v_1 and v_2 belonging to two perceptrons. These can be expanded in terms of the biases, synaptic weights and incoming data at each perceptron summation symbol. We get,

$$v_1 = \sum_{i=1}^{m_1} w_{1i} x_{1i} + b_1 \tag{1}$$

and

$$v_2 = \sum_{i=1}^{m_2} w_{2i} x_{2i} + b_2. \tag{2}$$

These are ordinary perceptrons; we are just considering two of them here. Next, consider them to be ephaptically coupled via coupling parameters $\lambda_{1,2}$. We generate two new variables representing the perceptrons, $\tilde{v}_1$ and $\tilde{v}_2$. In terms of the original variables v_1 and v_2, these can be written:

$$\tilde{v}_1 = v_1 + \lambda_2 v_2 \tag{3}$$

and

$$\tilde{v}_2 = v_2 + \lambda_1 v_1. \tag{4}$$

Now if $\lambda_{1,2} > 0$, we are in the case of normal ephaptic coupling. However, if these parameters can also be non-positive, then we are in the case of differential ephaptic coupling (Chawla & Morgera, 2022b). The negativity is allowed by the relaxation of electroneutrality of the bounding tract. These considerations extend in a straightforward way to $k > 2$ coupled perceptrons. However, in that case, the second additive term in Equations (3) and (4) will be as follows:

$$v_{eph} = \lambda_{ki} v_i \tag{5}$$

where the Einstein notation is used and both i and k run over the indices $1, ..., N$. This gives the individual perceptron as

$$\tilde{v}_k = v_k + \sum_i \lambda_{ki} v_i. \tag{6}$$

4 Conclusion

In this paper we proposed a new type of perceptron, inspired by ephaptic coupling in the mammalian nervous system. The vector ephaptic perceptron is constructed from two or more component perceptrons, whose variables are coupled to one another. The construction is based on the ephaptic current term of Reutskiy et. al. (Reutskiy *et al.*, 2003). By allowing for negative coupling between perceptrons, we extended the insight of (Chawla & Morgera, 2022b) to the machine learning setting.

The form of Equation (6) suggests that the model also allows for geometric considerations. Specifically, we might be able to model neural networks in a three dimensional setting. An advantage of such a formulation would be to mimic the gravitational wave considerations of (Chawla & Morgera, 2022a) in the perceptron context.

One of the limitations of the present work is a sound justification for the coupling. While coupled systems are found widely in nature and in the brain as well, and coupling is known to enable observed phenomena such as synchronization, whether such coupling would exist between neural cell bodies themselves, remains an open question. It is to be noted here that this paper differs from axon-axon coupling models in this regard.

In future work we hope to conduct simulations of the proposed model which will also illuminate whether there is a performance gain in using coupled perceptron networks instead of single perceptrons. For example, in the field of classification using neural networks, a model in Python can be generated and its convergence rate can be compared in the two cases - presence and absence of coupling. Geometric features can be investigated in a similar manner. Overall, the present paper expands our repository of neural network model systems that may perform better, even though the biological basis remains somewhat unclear.

References

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